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# Byzantine *Rechenbücher* An Overview with an Edition of *Anonymi* J and L[*]

Abstract: This article presents an overview of Byzantine *Rechenbücher* and an edition of two of them, earlier than any other published *Rechenbuch*. Along with the edition, a translation and a commentary are provided, as well as a complete thematic Greek-English glossary and an edition of the earliest known Byzantine table of decomposition of common fractions into unit fractions.

Keywords: Byzantine Mathematics, Byzantine *Rechenbücher*, Codex Laurentianus Plut. 86.3, Codex Parisinus suppl. gr. 387

Byzantine mathematics is “sectional” in its essence: it mainly comprises works that do not display a tight deductive structure. As a consequence, these works can easily be—or actually are—partitioned into independent sections, or can easily be assembled to generate sectional texts. Examples are logistic and geometric metrological writings, primers of any kind (including those to special astronomical “texts” like the *Persian Tables*)[1], scholia, isagogic compilations, compendia like the *Quadrivia*. Even such complex architectures as Metochites’ *Abridged Astronomical Elements* and Meliteniotes’ *Three Books on Astronomy* are sectional writings; a notable exception is Barlaam’s *Logistic*[2]. An extreme example of sectional mathematics are the so-called *Rechenbücher*, by no means a Byzantine speciality but a mathematical literary genre amply practised within the entire Mediterranean basin; nevertheless, fine specimens of this genre were produced in the Byzantine world.

Because of their highly sectional nature, to define what *Rechenbücher* are is a difficult task. We may say that they are collections of computational techniques and of arithmetical or metrological problems unrelated to each other, sometimes in (fictitious) daily-life guise[3], sometimes organized in sequences of almost identical items, and often formulated in a debased algorithmic code[4]. As a matter of fact, the “mathematical content” of a typical *Rechenbuch* problem is frequently related more to theoretical arithmetic (our number theory) than to logistic[5], the latter being the branch of arithmetic

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* I shall use the following bibliographic sigla in addition to the sigla currently used in JÖB: *DOO* = P. Tannery (ed.), Diophanti Alexandrini opera omnia cum Graeciis commentariis. I–II. Lipsiae 1893–1895; *HOO* = J. L. Heiberg – L. Nix – W. Schmidt – H. Schöne (eds.), Heronis Alexandrini opera quae supersunt omnia. I–V. Lipsiae 1899–1914. Online reproductions of almost all manuscripts mentioned in this article can be found by suitably searching the website https://pinakes.irht.cnrs.fr/. I thank Jens Høyrup for a fruitful discussion.

1 In the case of primers to tables, their sectional nature is obviously motivated by the nature of the reference text.

2 Study, (partial) edition, and discussion of the manuscript tradition of the mentioned treatises in B. Bydén, Theodore Metochites’ Stoicheiosis Astronomike and the Study of Natural Philosophy and Mathematics in Early Palaiologan Byzantium (*Studia Graeca et Latina Gothoburgensia* 66). Göteborg 2003; R. Leurquin (ed.), Théodore Méliténiote, Tribiblos Astronomique. Livre I; Livre II (*Corpus des Astronomes Byzantins* 4–6). Amsterdam 1990–1993; P. Carelos (ed.), Βαρλαὰμ τοῦ Καλαβροῦ, Λογιστική. Barlaam von Seminara, Logistiké (*Corpus philosophorum Medii Ævi. Philosophi byzantini* 8). Athens – Paris – Bruxelles 1996.

3 I put “fictitious” in brackets since some kinds of problems do answer to practical exigencies: these are problems on the calculation of interest or on equivalence of units of measurement (currency, weight, capacity). I use “problem” in the wide sense of a short, self-contained mathematical unit that (explicitly or implicitly) contains a series of operations devised to answer a specific question.

4 See pages 9–11 for a description of this stylistic resource.

5 It is not even said that any such “typical” texts exist: the 100 problems in the *Rechenbuch* I shall call *Anonymus* V are dis-

in which a unit can be divided and that deals with counting numbers and with calculations on them[6], for some *Rechenbuch* problems (but definitely not all of them) can be rewritten as Diophantine problems—that is, as algebraic equations. Still, the stylistic code of reference adopted in *Rechenbücher* suggests categorizing them within logistic. A genre with partly similar characteristics comprises arithmetical riddles in the form of epigrams, collected in part of Book XIV of the *Palatine Anthology*[7]. My definition of a *Rechenbuch* is a restrictive one: for instance, neither Planudes' *Great Calculation According to the Indians* and its anonymous 1252 source[8], nor Rhabdas' *Letter to Khatzykes* and its anonymous source[9], are *Rechenbücher* but computational primers; no *Quadrivium* is a *Rechenbuch* but it may contain both theoretical arithmetic in the style of Euclid, Nicomachus, or Diophantus, typically constituting the whole of the arithmetical part, and a computational primer, embedded in the astronomical part[10]; no computational primer in the style of the *Prolegomena ad Almagestum* (see the end of the next section) is a *Rechenbuch*.

The present article presents an overview of Byzantine *Rechenbücher* and an edition of two of them[11]. The meaning of "edition" in this case also deserves a clarification. *Rechenbücher* are, in fact, a kind of text that escapes standard philological methods for establishing filiations among manuscript witnesses: like any highly sectional text, such collections of disparate arithmetical problems can be assembled and de-assembled very easily, and any such problem is conducive to undergoing (major) variant readings in the process of transmission. Thus, hypotheses of filiation between versions of specific problems in different manuscripts cannot usually be corroborated by any uncontroversial textual evidence. The only sensible attitude is to edit every *Rechenbuch* separately[12], even when there are—as there frequently are—overlaps with other collections of the same kind. This is exactly the case with *Anonymus* L, published here, since it shares 24 problems out of 48 with the *Rechenbuch*, contained in the manuscript Par. suppl. gr. 387 and published by K. Vogel in 1968, which I shall call *Anonymus* P.

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tributed by the editors among 32 categories. To make categorizations of genres even more complex, recall that, within the doctrinal framework of the Neoplatonic author of the isagogic *prolegomena* to Nicomachus' *Introductio arithmetica*, the difference between theoretical arithmetic (Nicomachus) and arithmetical zetetic (Diophantus) lies in the polarity ἀριθμὸς μετρῶν / μετρούμενος "measuring / measured number" (*DOO* II 73.20–74.2). See the next section for the denominations I shall adopt in this article. The principle I have followed in assigning the denominations is to make the word *Anonymus* followed by a date if any such temporal determination figures in the text, and otherwise by a majuscule letter pointing to the library that preserves the manuscript containing the *Rechenbuch*. Of course, there are *Rechenbücher* that are not anonymous.

6 The best introduction to Greek logistic is still K. Vogel, Beiträge zur griechischen Logistik. Erster Teil (*Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Mathematisch-naturwissenschaftliche Abteilung*). Munich 1936, 357–472.

7 Scholia to some of these epigrams, presenting solutions to them, are edited by Tannery in *DOO* II 43–72, drawing from Paris, Bibliothèque nationale de France, supplément grec 384 (early–middle 10th century). On the structure of the collection see P. Tannery, Sur les épigrammes arithmétiques de l'Anthologie palatine. *REG* 7 (1894) 59–62, repr. Id., Mémoires scientifiques II. Toulouse – Paris 1912, 442–446, and further below.

8 The former is edited in A. Allard (ed.), Maxime Planude, Le grand calcul selon les Indiens. Louvain–la–Neuve 1981, the latter in A. Allard, Le premier traité byzantin de calcul indien: classement des manuscrits et édition critique du texte. *RHT* 7 (1977) 57–107.

9 See notes 64 and 65 below.

10 See for instance the computational primer for the sexagesimal system in §§ 1–6 and 26 of the astronomical part of Pachymeres' *Quadrivium*, in P. Tannery (ed.), Quadrivium de Georges Pachymères (*StT* 94). Città del Vaticano 1940, 330.33–363.11 and 451.15–454.16. My typology is further developed in F. Acerbi, Arithmetic and Logistic, Geometry and Metrology, Harmonic Theory, Optics and Mechanics, in: A Companion to Byzantine Science, ed. S. Lazaris. Leiden 2020, 105–159.

11 The German denomination is reminiscent of Latin *liber abbaci*, whose eponymous specimen is Fibonacci's (at least two versions, the latest one written in 1228).

12 Obvious exceptions to this philological stance must occur in those (very rare) cases in which a whole *Rechenbuch* is simply copied from one manuscript to another: this has happened with *Anonymus* P, copied in the manuscript El Escorial, Real Biblioteca del Monasterio de S. Lorenzo, Φ.I.16 (gr. 194), ff. 95r–115v, by John Mauromates (*RGK* I, no. 171; II, no. 229; III, no. 283) in March 1548.

Despite this extensive overlap, there are several reasons for publishing *Anonymus* L, which is contained in the manuscript Firenze, Biblioteca Medicea Laurenziana, Plut. 86.3, the main witness of Iamblichus' writings: as we shall see, it was almost certainly copied before *Anonymus* P. Some of the 24 problems in common with *Anonymus* P are nearly identical, but some display substantial variants: in general, the title system of *Anonymus* L is better structured, procedures and proofs are more detailed and calculations with fractions are worked out more explicitly and more thoroughly than in *Anonymus* P. I shall not enter into the details of these variants: a complete textual comparison of the problems *Anonymus* L shares with *Anonymus* P and with other similar writings would result in an overwhelming pile of minutiae. *Anonymus* P is not the only *Rechenbuch Anonymus* L shares problems with, in fact—and this just corroborates the philological point I made in the previous paragraph.

As a support to my edition, I shall also publish a (fragment of a) *Rechenbuch* contemporary with *Anonymus* L—these are six problems found on one single page of Città del Vaticano, Biblioteca Apostolica Vaticana, Vaticanus graecus 191, and which I shall call *Anonymus* J—and a complete list of resolutions of common fractions into unit fractions, with denominations running from 5 to 20, found in Par. gr. 1670 (an *Ur-Rechenbuch* I shall call *Anonymus* 1183).

The plan of the article is as follows. An overview of Byzantine *Rechenbücher* is followed by an explanation of the structure of "typical" *Rechenbuch* problems and of the stylistic code adopted in them. After this, the manuscript in which *Anonymus* L is transcribed, the mathematical contents of this collection, and a list of the resolutions of common fractions into unit fractions used in the text are presented. The subsequent section briefly sets out the contents of *Anonymus* J and its salient stylistic features. A thematic word index of the edited texts follows. After some information preliminary to the edition, the edition itself is provided; every problem is followed by a translation and, in most cases, by a commentary. In the Appendix, the list of resolutions of common fractions into unit fractions in Par. gr. 1670 is transcribed and translated in tabular form; it is followed by a specimen of the method apparently used to find any of these resolutions.

## BYZANTINE *RECHENBÜCHER*: AN OVERVIEW

The *Rechenbücher* I know of are set out in the following list[13].

*Anonymus* 1183, Par. gr. 1670 (end 12th century), ff. 21v–61v[14]. This is something like an *Ur-Rechenbuch*, namely, a collection of apparently disconnected subsets of problems. It contains:

[13] On the phenomenon of *Rechenbuch*-style problems attached to logistic treatises, see F. Acerbi, I problemi aritmetici attribuiti a Demetrio Cidone e Isacco Argiro. *Estudios bizantinos* 5 (2017) 131–206: 176–177, and Acerbi, Arithmetic 134. Even if chronology might suggest including the Papyrus Achmin and the relevant epigrams of *AP* XIV in the list, their location and form of transmission suggest to me that they should be regarded as products of Late Antiquity. See below for the contents of these documents.

[14] The manuscript is described in *HOO* IV x–xi (with edition of the text at f. 61v ibid., xvii); F. Acerbi – B. Vitrac (eds.), Héron d'Alexandrie, Metrica (*Mathematica Graeca Antiqua* 4). Pisa – Roma 2014, 436–437; F. Acerbi, Struttura e concezione del vademecum computazionale Par. gr. 1670. *Segno e Testo* 19 (2021), in print, with a complete "translation" of the list of multiples of currency units, and an edition of the list of submultiples and of the Easter Computus. Edition of ff. 3r–21v in B. de Montfaucon – J. Lopin – A. Pouget, Analecta Graeca. Lutetiae Parisiorum 1688, 316–392; Montfaucon also used this material for the chapters on technical abbreviations in his celebrated *Palaeographia Graeca*. Parisiis 1708, 359–367. These folia of Par. gr. 1670 contain the treatises of fiscal accounting known as *Palaia Logarikê* (ff. 3r–13r) and *Nea Logarikê* (13r–21v), composed shortly after the death of Alexios I Komnenos in 1118; most accessible complete edition in C. E. Z. von Lingenthal, Jus Graeco-Romanum, Pars III, Novellae constitutiones. Lipsiae 1857, 385–400 (resorting to a tabular set-up that destroys the original layout); commentaries in M. F. Hendy, Coinage and Money in the Byzantine Empire 1091–1261 (*DOS* 12). Washington DC 1969, 50–64, and C. Morrisson, La *logarikè*: Réforme monétaire et réforme fiscale sous Alexis Ier Comnène. *TM* 7 (1979) 419–464 (with complete French translation). Edition of *Anomymi* 1183, 1256, 1306, and R in F. Acerbi, Byzantine Logistic Texts. forthcoming.

ff. 21v–34v, multiples and submultiples of currency units[15]; 35r–46v, a detailed collection of procedures for dividing numbers 1 … *n* by *n*, with *n* = 5 … 12, followed (44v–46v) by a list of the mere results of the same divisions, ranging this time from 5 to 20 (this list is edited in the present article); 46v–61v, Easter Computus and other chronological material[16], repeatedly assuming a.m. 6691 [= 1183] as the current year; 61v, measure of a stone solid. Greek numerals are used. The final part of the manuscript (ff. 62r–130v) contains geometric metrological material[17].

*Anonymus* E, Scorial. X.IV.5, gr. 400 (13th century); 259 items (entire manuscript), without a title. It includes standard riddles, applications of the rule of three, and Diophantine-style problems in everyday-life guise, problems of conversion involving weight and currency units of measurement, calculations with fractions. In the Cypriot vernacular language. The style and specific contents obviously relate this item to the following one. Greek numerals are used.

*Anonymus* 1256, Vat. Pal. gr. 367 (1317–20), ff. 69r–97v[18]. The style displays a slight tinge of vernacular Greek. Its contents include: ff. 69r–83v, title μέθοδοι σὺν θεῷ λογαρικοὶ ὡς ἐν ἐπιτόμῳ πάνυ ὠφέλιμοι τοῖς νουνεχῶς προσέχουσιν αὐτοὺς νέοις *Abridged Computational Procedures Very Useful for the Young People Carefully Attending Them*, 109 items featuring standard riddles, applications of the rule of three, and Diophantine-style problems in everyday-life guise (the riddle of the ring opens the collection), problems of conversion involving weight and currency units of measurement, calculations with fractions; 83v–84r, a table of decomposition of common fractions into unit fractions, set out as usual as division of numbers 1 … *n* by *n*, with *n* = 6 … 17; only one resolution is set out; 84v, standard Easter table; 85r–88r, Easter Computus and other chronological material, assuming a.m. 6764 [= 1256] as the current year; 88v–92v, capacity of ships and measurement of quantities of specific goods like oil, wine, and salt; 92v–93v, two testament templates; 94r–97v, geometric metrological problems[19]. Greek numerals are used.

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15 Titles ἀρχὴ σὺν θεῷ τῶν λιτρισμῶν *Beginning, with God, of the Measures by Pounds*, and περὶ τῶν λεπτῶν τῆς λίτρας *On the Parts of the Pound*, at ff. 21v–33v and 33v–34v, respectively. The units involved are 1 κεντηνάριον = 100 λίτραι = 7200 νομίσματα, the latter being identified with the ἑξάγιον (see note 56 below).

16 One must bear in mind that the traditional denomination "Easter Computi" for such chronological primers frequently amounts to a categorial mistake, as the computation of the Easter date was only the main goal of a whole system of tightly interrelated chronological issues. The Byzantine tradition of chronological primers, which developed independently of the tradition of *Rechenbücher* {early example (on f. 4v, the assumed current year is a.m. 6400 [= 891/2]) e.g. in Par. suppl. gr. 920 (10th century), ff. 2r–17r: on this manuscript see now F. Acerbi, How to Spell the Greek Alphabet Letters. *Estudios bizantinos* 7 (2019) 119–130}, has not yet been explored in a systematic way: O. Schissel, Note sur un Catalogus Codicum Chronologorum Graecorum. *Byz* 9 (1934) 269–295; recent editions and studies include A. Tihon, Le calcul de la date de Pâques de Stéphanos-Héraclius, in: Philomathestatos. Studies in Greek and Byzantine Texts Presented to Jacques Noret for his Sixty-Fifth Birthday, ed. B. Janssens – B. Roosen – P. Van Deun (*Orientalia Lovaniensia Analecta* 137). Leuven 2004, 625–646; J. Lempire, Le calcul de la date de Pâques dans les traités de S. Maxime le Confesseur et de Georges, moine et prêtre. *Byz* 77 (2007) 267–304; A. Tihon, Barlaam de Seminara. Traité Sur la date de Pâques. *Byz* 81 (2011) 362–411.

17 Edited by Heiberg in *HOO* IV. On the criteria followed by Heiberg in his edition of the Greek geometric metrological corpus, resulting in two philological monsters, see Acerbi – Vitrac, Héron d'Alexandrie 430–433.

18 This important manuscript is the paradigmatic example of the script type called "chypriote bouclée": P. Canart, Un style d'écriture livresque dans les manuscrits chypriotes du XIVe siécle: la chypriote "bouclée", in: La paléographie grecque et byzantine. Actes du Colloque Paris, 21–25 octobre 1974, ed. J. Glénisson – J. Bompaire – J. Irigoin (*Colloques internationaux du C.N.R.S.* 559). Paris 1977, 303–321, repr. Id., Études de paléographie et de codicologie. I (*StT* 450). Vatican City 2008, 341–359. Analysis of the manuscript, including several datings occurring in it, in A. Turyn, Codices graeci Vaticani saeculis XIII et XIV scripti annorumque notis instructi. Vatican City 1964, 117–124 and pl. 96.

19 The metrological problems are edited in E. Schilbach, Byzantinische metrologische Quellen (*Byzantina keimena kai meletai* 19). Thessalonike 1982, sects. I.5c–d (ff. 98r and 80v); II.4, 14, 16, 18 (ff. 94r–97v); III.1 (ff. 88v–91r); III.2e,k (f. 73r27–v9); IV.4d (ff. 80r23–v4, 83v marg.); IV.8b,f (88v1–3, 84r marg., 76v16–19, 69v5–9); see also ibid., 13; and in J. Lefort – R.-C. Bondoux – J.-C. Cheynet – J.-P. Grélois – V. Kravari – J.-M. Martin, Géométries du fisc byzantin (*Réalités byzantines* 4). Paris 1991, 48–58 (ff. 94r–97v). The two testament templates are edited in G. Ferrari, Due formule notarili cipriote inedite del Cod. Vaticano Pal. gr. 367, in: Studi in onore di Biagio Brugi nel XXX anno del suo insegnamento. Palermo 1910, 429–443.

*Anonymus* L, Laur. Plut. 86.3, ff. 165r–169v (2nd half of 13th century); 48 items partitioned into subsections. Greek numerals are used. This is edited and analysed in the present article.

*Anonymus* J. Vat. gr. 191, f. 261r (2nd half of 13th century); 6 items, title ἀρχὴ σὺν θεῷ τῶν διαφόρων ἐρωτημάτων. This single page, deleted by pen strokes, is embedded into an astrological collection: the bifolio where these problems belong was thus used as recycled paper. This is also edited and analysed in the present article.

*Anonymus* P, Par. suppl. gr. 387, ff. 118v–140v (end 13th century); 119 items, title ψηφηφορικὰ ζητήματα καὶ προβλήματα, ἃ δὴ καὶ μετὰ τῶν οἰκείων μεθόδων ἕκαστον σύγκειται *Calculative Investigations and Problems, Which Are Collected here Each with its Own Procedures, too*[20]. It also contains some geometric metrological problems and number-theoretical elaborations. The distribution of the problems among categories is random. Greek numerals are used. Most of what precedes in the manuscript is isagogic or geometric metrological material[21].

*Anonymus* 1306, Par. suppl. gr. 387, ff. 148r–161v (early 14th century). This is also something like an *Ur-Rechenbuch*. Its contents are: ff. 148r–149v, operations on fractions; 149v, abridged Passover Computus (from a given year to the subsequent one) to a.m. 6814 [= 1306], and other chronological material; 150r–151r, very short annotations (one of which is dated 1303), followed by one *Rechenbuch*-style problem; 151v, Eratosthenes' sieve; 152r–v, calculation of currency interests, title ἑτέρα ψηφιφορία περί τε τόκων νομισμάτων διαφορᾶς τε καὶ φυρασίας, καὶ ἔστιν εἰπεῖν οὕτως περὶ τόκων νομισμάτων *Calculation about the Difference and Combination of Interests of Nomismata, Which Amounts to Say about Interests of Nomismata*; 152v–157r, basic applications of the rule of three, title ἑτέρα μέθοδος ἀριθμητικὴ περὶ κέρδους καὶ ζημίας *Another Arithmetical Procedure about Profit and Loss*; 157r–158r, rules for calculating with unit fractions, title ψηφιφορία περὶ συνθέσεως μορίων ἐκβολῆς διαιρέσεώς τε καὶ πολλαπλασιασμοῦ *Calculation about Addition, Subtraction, Division and Multiplication of Parts*; 158r–161v, three sets of typical *Rechenbuch*-style problems: first set, 8 items, no title[22]; second set, 4 items, title ψηφιφορικὰ προβλήματα πάνυ ὀφέλημα *Very Useful Calculative Problems*[23]; third set, 6 items, title μέθοδοι καθολικαί *General Procedures*.

Rhabdas, *Letter to Tzavoukhes*[24]. Embedded in a discursive setting the other *Rechenbücher* do not share, it contains: multiplication and division (by reduction) of unit fractions (118.1–126.29 in

[20] Edition K. Vogel (ed.), Ein byzantinisches Rechenbuch des frühen 14. Jahrhunderts (*WBS* 6). Vienna 1968. The manuscript is described in *HOO* IV iv–vii; M.-L. Concasty, Un manuscrit scolaire (?) de mathématiques. *Scriptorium* 21 (1967) 284–288; Acerbi – Vitrac, Héron d'Alexandrie 437–439. *Anonymus* 1306 is in a hand different from (and later than) that of *Anonymus* P (A. Gioffreda, *per litteras*). Thus it is incorrect, as Concasty, Un manuscrit 285, and Vogel, Ein byzantinisches Rechenbuch 11 n. 1a, do, to assign the date of the former to the latter.

[21] Edited in *HOO* IV–V, with the same warning as above. The isagogic material is the pseudo-Heronian *Definitiones*. Ff. 141r–147v contain extracts from the arithmetical section of the so-called *Anonymus Heiberg*—J. L. Heiberg (ed.), Anonymi Logica et Quadrivium (*Det Kongelige Danske Videnskabernes Selskabs, Historisk-filologiske Meddelelser* 15.1). Copenhagen 1929, sects. 5–8, 52.3–54.6; 10–12, 54.23–55.1, 55.10–15, 55.17–24; and 21, 62.12–19, the latter in a later hand—but Heiberg did not use this manuscript—and (at ff. 142v–147r) a description of a cosmological system.

[22] The first item of this subset is also attested, followed by a solution, in Par. gr. 2107, f. 129v (end 14th–beginning 15th century), title αἴνιγμα ψηφικόν; both are edited in Acerbi, I problemi aritmetici, Text 16 (and n. 110 for commentaries on the variants involved). The riddle can already be found in *AP* XIV.51.

[23] The first three items of this subset coincide with the first three in *Anonymus* L, the first two also coincide with nos. 62 and 63 of *Anonymus* P. Later in the manuscript, ff. 181v–208r contain a substantial collection of problems on conversion of units of measurement, a lore a title dubs νοταρικὴ ἐπιστήμη "notarial knowledge". On f. 209r–v, title ἀρχὴ σὺν θεῷ τῶν παραπέμπτων, a procedure for computing the inverse of superparticular ratios (from $\frac{4}{5}$ to $\frac{9}{10}$) of integer numbers, followed by a list of such ratios.

[24] Edition P. Tannery, Notice sur les deux lettres arithmétiques de Nicolas Rhabdas. *Notices et extraits des manuscrits de la Bibliothèque Nationale* 32 (1886) 121–252, repr. Id., Mémoires scientifiques IV. Toulouse – Paris 1920, 61–198: 118–186, but two problems at the end are omitted because they were already edited in R. Hoche (ed.), Nicomachi Geraseni pythagorei Introductionis Arithmeticae libri II. Lipsiae 1866, 152.4–154.10. The main manuscript witnesses are organized as follows:

Tannery's edition); two methods of extraction of an approximate square root, the one a refinement of the other (128.1–134.22); Easter Computus, assuming a.m. 6849 [= 1341] as the current year (134.23–138.28); a so-called μέθοδος πολιτικῶν λογαρισμῶν *Procedure of Civil Life Computations*, namely: an exposition of the several species of the rule of three (140.1–144.9); generalities and some problems of conversion involving weight[25], length, and currency units of measurement, solved by application of the previous rules (144.10–154.5); the same for a problem involving alloying (154.6–24); twenty *Rechenbuch*-style problems[26], with solutions and associated procedures (156.25–186.19). Greek numerals are used.

*Anonymus* 1436, Vindob. phil. gr. 65, ff. 11r–126r (15th century); 242 numbered items[27]. The manuscript contains, in the margins and within the text but always in the hand of the main copyist, hundreds of completed arithmetic operations. In two books (nos. 1–116 and 117–242), written in vernacular Greek, with obvious lexical loans from Italian and Arabic-Turkish. It includes a fragmented handbook of logistic featuring notational issues, including the sign for zero (nos. 1–5); multiplication (with an example assuming 1436 as the current year) and division of integers (6–39); operations with fractions (40–52); extraction of an approximate square root by linear interpolation (123); extraction of cube roots (118); calculations with roots (119–126, 128–133); standard multiplication tables (no. 127 = ff. 67v–73r; ff. 118r–123v contain square roots tables, empty for the most part). Apart from this, one finds rule of three and arithmetical problems (nos. 53–116, 153–165), sometimes without the daily-life guise (134–152)[28], and including the sum of arithmetic progressions (57–60); geometric problems solved numerically and geometric metrological problems (166–242)[29].

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Par. gr. 2107, ff. 115v–122v (Tannery, Notice 140.1–172.15 πολυπλασίασον {ταῦτα}; 1425–48), copies of which are Wien, Österreichische Nationalbibliothek, suppl. gr. 46 (<George Valla>), ff. 1r–4r, and Wolfenbüttel, Herzog-August-Bibliothek, Gud. gr. 40 (<Matthew Macigni>), ff. 2r–8r; Vat. gr. 1411 (<John Eugenicus>), ff. 23r–25v (incomplete, *des.* ibid., 132.31 ἐστιν ὁ κε); its apographs are Scorial. Φ.I.10 (gr. 188), ff. 108v–124r (1542), an immediate copy of which is Par. gr. 2428, ff. 225r–245v (mid-16th century), Vat. Ross. 986 (mid-15th century), ff. 123r–141v, Par. suppl. gr. 652 (15th century), ff. 165r–v (*des.* ibid., 122.11 τρισκαιδέκατα). On all of these manuscripts see Acerbi, I problemi aritmetici; add also Par. suppl. gr. 682, f. 34r–v (15th century), containing only the Easter Computus. See P. Tannery, Manuel Moschopoulos et Nicolas Rhabdas. *Bulletin des Sciences mathématiques* 2e série 8 (1884) 263–277, repr. Id., Mémoires Scientifiques IV. Toulouse – Paris 1920, 1–19: 12–14, for a summary description of the contents of the treatise. On this Easter Computus (a real Computus, not a chronological primer) see O. Schissel, Die Osterrechnung des Nikolaos Artabasdos Rhabdas. *BNJ* 14 (1937–38) 43–59.

25 The metrological portion at Tannery, Notice 144.11–146.8, is also edited in Schilbach, Byzantinische metrologische Quellen, sect. IV.3; see also ibid., 30–31.

26 Some of these problems coincide with problems in *Anonymus* P: no. 13 = example at Tannery, Notice 142.26–144.9; no. 14 = Rhabdas' problem I; 18 = problem III; 20 = IV; 21 = VI; 22 = VII; 9 = X; 11 = XII; 24 = XIII; 35 = XVI. Algebraic formulations of the problems in this section are in Tannery, Manuel Moschopoulos 14. The title of this section returns in the phrases at Tannery, Notice 140.8 and 154.3–4.

27 Editions: Books I–II, M. D. Chalkou (ed.), The Mathematical Content of the Codex Vindobonensis Phil. Graecus 65 (ff. 11–126). Introduction, Edition and Comments (*Byzantine Texts and Studies* 41). Thessaloniki 2006; Book I, S. Deschauer (ed.), Die große Arithmetik aus dem Codex Vind. phil. gr. 65. Eine anonyme Algorismusschrift aus der Endzeit des Byzantinisches Reiches. Textbeschreibung, Transkription, Teilübersetzung mit Fachsprache, Vokabular, Metrologie (*Österreichische Akademie der Wissenschaften, Philosophisch-historische Klasse, Denkschriften* 461). Vienna 2014. Other texts pertaining to the logistic part of this item are found on ff. 4v–5v, 6r–9v and 142v–159v of the manuscript; the latter mainly repeat sections of *Anonymus* 1436. A tract, explicitly presented as a complement to Nicomachus, written by the Aristotelian commentator Leo Magentinus (1st half of 14th century; *PLP*, no. 16027) and entitled Περὶ τοῦ πῶς ἐστιν ὁ δέκα τέλειος ἀριθμός O*n Why Ten is a Perfect Number*, is also found in Vindob. phil. gr. 65, f. 4r–v. Related material can be found at ff. 1v–2v and 5v–6r of the same manuscript (one text is transcribed twice, the former being the better version). For a description of this manuscript, H. Hunger, Katalog der griechischen Handschriften der Österreichischen Nationalbibliothek, I (*Museion* 4.1). Vienna 1961, 182–183, must be completed with Deschauer, Die große Arithmetik 11*–12*

28 Thus, these are algebraic problems in Diophantine style and worded in the typical Middle-Ages fashion (the unknown is called πρᾶγμα, etc.). This feature is unique to *Anonymus* 1436.

29 Note that nos. 185–200 are missing because a page was lost in some model of Vindob. phil. gr. 65 (which does not show traces of a missing page); their content (mainly rules for fortification-building) can be recovered from the initial table of

*Anonymus* V, again Vindob. phil. gr. 65 (15th century), ff. 126v–140r; 100 numbered items[30]. Written in vernacular Greek, with obvious lexical loans from Italian and Arabic-Turkish. It also contains a few computational methods and some metrological problems. *Anonymi* 1436 and V only use Greek numerals, with an additional figure for the zero; sometimes, the Greek numeral signs from α to θ are also used to designate tens, hundreds, etc.: the resulting notation is positional.

*Anonymus* R, Firenze, Biblioteca Riccardiana, gr. 12, ff. 26v–27r (1430–50); 6 items[31].

*Anonymus* U, Uppsala, Universitets Bibliotek, gr. 8 (late 15th century), ff. 324r–331r; 18 items[32]. Written in vernacular Greek, with obvious lexical loans from Italian. Twelve problems are followed by six exercises on multiplication and division of fractions. Both Greek and Western Arabic numerals are used.

Add to these items a florilegium of geometric metrological problems, some of which are in fact problems of Diophantine analysis in fictitious metrological guise (problems "of separation"), contained in Istanbul, Topkapı Sarayı Mūzesi G.İ.1 (written by Ephrem ca. 960), ff. 28v–38v[33].

The descriptions of some of the above items confirm that the designation *Rechenbuch* must be taken to refer to a constellation of more or less well-structured, highly sectional, logistic collections; these can sometimes prove difficult to delimit in a given manuscript, because of the simultaneous presence of geometric metrological material that we might wish to attach to the intended *Rechenbuch* or not.

The existence of what I have called *Ur-Rechenbücher* adds a diachronical dimension to the issue: we really see the generation of these corpora from core collections of metrological recipes (conversions of weights and currencies, but also measurement of geometric figures) accompanied by computational tools obviously relevant for solving these problems such as resolution of common fractions into unit fractions. It is noteworthy that the chronological primers traditionally called Easter Computi were included in (*Ur*-)*Rechenbücher* from the very outset: apparently, they were perceived as homogeneous material in point of style and insofar as they involve extensive calculations. Problems in fictitious daily-life guise seem to enter the corpus during the Nicaean period (1204–61), thereby giving rise to fully-fledged *Rechenbücher*. Now, it so happens that: *a*) these problems have a longstanding Greek tradition in the form of epigrams (*AP* XIV)[34]; *b*) a purely mathematical setting for

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contents (f. 13r–v); no. 117 is the preface to Book II.

30 Edition H. Hunger – K. Vogel (eds.), Ein byzantinisches Rechenbuch des 15. Jahrhunderts (*Österreichische Akademie der Wissenschaften, Philosophisch-historische Klasse, Denkschriften* 78.2). Vienna 1963; the copyist is not the same as that of *Anonymus* 1436. The manuscript was first described, with an edition of some extracts, in J. L. Heiberg, Byzantinische Analekten. *Abhandlungen zur Geschichte der Mathematik* 9 (1899) 163–174: 163–169; among these extracts (ff. 146v–147r) figures a numerical list of powers of 2 as far as $2^{63}$, with three additional texts (a rule for getting the sum as far as an arbitrary power, a rule for multiplying specific powers, a note on some peculiar denominations of higher numerical orders; only the latter is edited by Heiberg): this is the so-called "wheat and chessboard problem"; the same copyist transcribed the list and two of the three texts in the manuscript Milano, Biblioteca Ambrosiana, I 112 sup. (gr. 469), ff. IIIv–IVr; a chessboard scheme in whose cells the same numbers are marked is in Ambros. E 80 sup. (gr. 294), f. 196v (the last two cells are empty). A problem identical with *Anonymus* V, no. 38, is edited in F. Spingou, Πῶς δεῖ εὑρίσκειν τὸ δακτύλιον. Byzantine Game or a Problem from Fibonacci's *Liber Abaci*? Unpublished Notes from *Codex Atheniensis* EBE 2429. *Byz* 84 (2014) 357–369, but the editor got all the mathematics wrong.

31 The second in order coincides with the one edited in Hoche, Nicomachi Geraseni 152.5–153.6, the third with the one included in Rhabdas' *Letter to Tzavoukhes* and edited in Tannery, Notice 184.20–186.4. All the problems were penned by George Scholarios (d. c. 1472; *PLP*, no. 27304—I thank D. Speranzi, who communicated the description of the manuscript in his forthcoming catalogue of the Greek manuscripts of the Riccardiana library to me).

32 Edition D. M. Searby, A Collection of Mathematical Problems in Cod. Ups. Gr. 8. *BZ* 96 (2003) 689–702.

33 See J. L. Heiberg – H. G. Zeuthen, Einige griechische Aufgaben der unbestimmten Analytik. *Bibliotheca Mathematica*, III Folge, 8 (1907–08) 118–134, and Acerbi – Vitrac, Héron d'Alexandrie 492–497. A *Rechenbuch* problem was also attached at the end of Planudes' *Great Calculation According to the Indians*; we read it in Allard, Maxime Planude 191.17–193.21; it is the same problem as *Anonymus* L, no. 40 = *Anonymus* P, no. 84.

34 A typology of the mathematical epigrams in *AP* XIV is as follows: partition with a remainder, that is, an unknown number

some of them is provided in Diophantus' *Arithmetica* and in a possibly lower-status tradition that surfaces in P.Mich. 620 (2nd century)[35]; *c*) finally and most importantly, Greek Late Antiquity hands an almost fully-fledged *Rechenbuch* down tu us as the Papyrus Achmin (7th century)[36]. These facts mean that it is open to question whether we have to assume that any *early and massive* transfer of lore and techniques of this kind from other mathematical cultures in the Mediterranean basin has occurred, in particular from the Latin world, to the Greek technical corpus[37]. Very simply—and despite the arguably contrary evidence of *Anonymi* E and 1256 coming from Cyprus—the early Greek *Rechenbuch* tradition is, on the whole, perfectly self-contained; for this reason, in my edition I shall only provide a concordance with similar problems in Greek sources[38]. Moreover, it is quite obvious that *Anonymi* L, J, P, and 1306 on the one hand, and *Anonymi* E and 1256 on the other, must relate to markedly homogeneous yet different campaigns of composition of this kind of collections.

It is also important to recall that the Greek and Byzantine scientific literature displays an independent tradition of strictly logistic primers intended to assist astronomical calculations[39]. These primers give theoretical grounds for, and explain how to perform, the basic arithmetical operations in the decimal or in the sexagesimal system, including extraction of approximate square roots and composition

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is the sum of given parts of itself and of a given number: 1–4, 116–127, 137, 138 (116–120, 138 on distributing nuts or apples; 126, 127 on telling the age; 126 tells the age of Diophantus); the sum of given parts of an unknown number is a given number: 50; an unknown number plus a given part of itself yields a given number: 6, 139–142 (telling the hour), 128, 129, 143 (various settings; the last with two given parts); filling of a tank: 7, 130–136; numbers in arithmetic progression with given ratio and sum, and unknown first term: 12; two or several unknown numbers satisfying specific relations: 11, 13, 48, 49, 51, 144 [the relations are 11, 13: $x + y = k$ and $x/a \pm y/b = h$; 48: $ax = n(a + k)$ ($n$ arbitrary; the solution is not unique); 49: $x + y + z + w = k$, $x + y = ck$, $x + z = bk$, $x + w = ck$; 51: $x = y + z/3$, $y = z + x/3$, $z = 10 + y/3$; on 51 see also note 22 above; 144: $z + w = x$, $2w = x$, $z = 3y$ (indeterminate)]; give-take problems: 145, 146. These epigrams and the scholia to them are edited together, from Par. suppl. gr. 384, in *DOO* II 43–72. See also Tannery, Sur les épigrammes, and P. Tannery, Le calcul des parties proportionnelles chez les Byzantins. *REG* 7 (1894) 204–208, repr. Id., Mémoires scientifiques IV. Toulouse – Paris 1920, 283–287, for an assessment. Recall that one single problem in Diophantus' *Arithmetica*, namely, V.33, is conceived as the solution of a riddle set out in epigram form.

35 Edition in F. E. Robbins, P. Mich. 620: A Series of Arithmetical Problems. *Classical Philology* 24 (1929) 321–329, further discussion in K. Vogel, Die Algebräischen Probleme des P. Mich. 620. *Classical Philology* 25 (1930) 373–375.

36 The Papyrus Achmin [edition J. Baillet, Le papyrus mathématique d'Akhmîn. *Mémoires publiés par les membres de la Mission Archéologique Française au Caire* 9.1 (1892) 1–89] contains 50 problems, sometimes very short. The typology is as follows (cf. ibid., 32–33): calculation of volumes: 1, 2, 5; proportional partition: 3, 4, 10, 11, 47–49 (the three treasures problem); iterative partition: 13, 17; calculation of interest: 26–28, 33–37, 44–46; basic rule of three: 41–43; calculations with fractions: 6–9, 12, 14–16, 18–25, 29–32, 38–40, 50. The problems are preceded by a table of resolutions of common fractions into unit fractions; see pages 14–15 and 50–56 below.

37 A similar claim concerning the *Rechenbuch* he publishes is made but not argued in Vogel, Ein byzantinisches Rechenbuch 154–160 and the all-inclusive table there attached. For a different assessment concerning *Rechenbücher*, see J. Høyrup, Fibonacci – Protagonist or Witness? Who Taught Catholic Christian Europe about Mediterranean Commercial Arithmetic? *Journal of Transcultural Medieval Studies* 1 (2014) 219–247: 236–238, who sees it as more likely a partial borrowing in the opposite direction, namely, "that the Italian and Iberian way to formulate alloying problems had its roots in a *Byzantine* money-dealers environment" (ibid., 238, emphasis in the original). Recall that Fibonacci claims three times that one of his problems was proposed to him by a *magister constantinopolitanus* (B. Boncompagni (ed.), Scritti di Leonardo Pisano. II. Liber abbaci. Rome 1857, 188, 190, 249). This is in fact the sole basis supporting the claim that Fibonacci was present in Constantinople at the end of 12th century.

38 The reader interested in concordances of problems in Greek and non-Greek sources will find them in Vogel, Ein byzantinisches Rechenbuch 154–160; Hunger – Vogel, Ein byzantinisches Rechenbuch 91–101; and, on a systematic basis and ranging over the entire worldwide corpus, in J. Tropfke, Geschichte der Elementarmathematik, 4. Auflage. Berlin–New York 1980, sect. 4.

39 Cf. the explicit statement opening *Anonymus* 1252: Allard, Le premier 80.2–4, and, in a smoother formulation, Planudes' *Great Calculation*: Allard, Maxime Planude 27.1–5. Despite its title (and the author's statement similar to that of Planudes: Carelos, Βαρλαὰμ 1.10–26), Barlaam's *Logistic* is not a writing of logistic, but a fully-fledged treatise of theoretical arithmetic formulated in impeccable demonstrative style. Barlaam (*PLP*, no. 2284), undisputably the Byzantine scholar best versed in mathematical matters and a major actor in the hesychastic controversy, died in 1348.

and removal of ratios[40]. The two traditions eventually merged in the 15th century, within *Anonymus* 1436, for instance. More generally, the later *Rechenbücher* appear to witness to a discontinuity in the tradition, entailing obvious stylistic changes: these involve contents (as just seen), lexicon (with obvious loans from other languages, in particular Italian), and the style in which the problems are written (less strict algorithmic code).

## GENERAL STYLISTIC FEATURES OF *RECHENBÜCHER*

A typical *Rechenbuch* problem is presented as a question (ἐρώτησις) or as a calculation (ψῆφος). The enunciation first sets out the givens and the constraints of the problem; the task to be performed is then enunciated in interrogative or prescriptive form[41]. The enunciation is followed by the procedure of solution (μέθοδος). The input of the procedure is fed in either by means of a causal subordinate ἐπειδή "since" + indicative, or directly within the first algorithmic step, after a standard initializing "we do as follows" clause. The procedure may be followed by a proof (ἀπόδειξις; they are absent in *Anonymus* J), which amounts to checking that the numbers arrived at at the end of the procedure actually solve the problem. The procedure and especially the proof may include elaborate calculations with fractions, usually not carried out in full details. These operations constitute the computational core of *Rechenbuch* problems; as we have seen, specific *Rechenbuch* problems just deal with manipulations of fractions. As was customary in the Greek tradition, common fractions were handled by resolving them into unit fractions (for instance, $^2/_7$ was resolved into $^1/_{12}$ $^1/_{22}$ $^1/_{33}$ $^1/_{44}$); these unit fractions are combined with the relevant ones featuring elsewhere in the problem, in order to add or to subtract the common fractions they arise from[42]. *Rechenbuch* problems other than geometric metrological problems usually do not involve the extraction of square roots.

The style adopted in *Rechenbuch* problems calls for some words of explanation. Greek and Byzantine mathematics adopted three stylistic codes: these are the demonstrative, procedural, and algorithmic codes[43]. The demonstrative code is the one in which ancient Greek geometry is written and does not need any description. In logistic, the solution of a numerical problem, usually formulated without any supporting proof, was encoded in two peculiar expository formats, namely, the procedural and the algorithmic code. These are two stylistic resources formulating chains of operations on numerical entities, such that the output of an operation is taken as the input of the subsequent operation: they are the ancient counterpart of our computer programmes. In particular, the procedural code was aptly used to formulate operational sequences that we would condense in an algebraic "formula".

The procedural code formulates its prescriptions as a sequence of coordinated principal clauses with the verb in the imperative or in the first person plural, present or future; to each principal clause are subordinated one or more participial clauses coordinated with each other; the participle is a satellite and performs the function of modifier of the operating subject. This code is used to formulate operatory prescriptions in the most general way; the mathematical objects involved are identified by

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[40] See the overview in Acerbi, Arithmetic 117–124. The model of such primers is the *Prolegomena ad Almagestum*, a (unredacted) set of lecture notes of a course held in the circle of the Neoplatonic philosopher Ammonius (Alexandria, end of 5th century); see J. Mogenet, L'Introduction à l'Almageste (*Académie Royale de Belgique, Classe des Lettres et des Sciences Morales et Politiques, Mémoires, 2e* série, Tome 51.2). Louvain 1956, and the edition of the non-logistic portion in F. Acerbi – N. Vinel – B. Vitrac, Les *Prolégomènes à l'Almageste*. Une édition à partir des manuscrits les plus anciens: Introduction générale – Parties I–III. *SCIAMVS* 11 (2010) 53–210. These primers usualy do not include instructions for handling common or unit fractions.

[41] Both directive infinitive and modal expressions are used; see the thematic word index below.

[42] See pages 14–15 and the Appendix for details.

[43] These notions were first introduced in F. Acerbi, I codici stilistici della matematica greca: dimostrazioni, procedure, algoritmi. *Quaderni Urbinati di Cultura Classica* 101.2 (2012) 167–214; see also Acerbi – Vitrac, Héron d'Alexandrie, sect. II.2, for the algorithmic code in Hero's *Metrica*.

(sometimes extremely long) definite descriptions; the verb forms—either finite or participial—represent the operations. The most striking application of this stylistic tool in the ancient Greek corpus is the double procedure in Diophantus, *De polygonis numeris*, of which we only read the first half as an example[44]:

> λαβόντες γὰρ τὴν πλευρὰν τοῦ πολυγώνου ἀεὶ διπλασιάσαντες ἀφελοῦμεν μονάδα, καὶ τὸν λοιπὸν πολλαπλασιάσαντες ἐπὶ τὸν δυάδι ἐλάσσονα τοῦ πλήθους τῶν γωνιῶν [καὶ] τῷ γενομένῳ προσθήσομεν ἀεὶ δυάδα, καὶ λαβόντες τὸν ἀπὸ τοῦ γενομένου τετράγωνον ἀφελοῦμεν ἀπ' αὐτοῦ τὸν ἀπὸ τοῦ τετράδι ἐλάσσονος τοῦ πλήθους τῶν γωνιῶν, καὶ τὸν λοιπὸν μερίσαντες εἰς τὸν ὀκταπλασίονα τοῦ δυάδι ἐλάσσονος τοῦ πλήθους τῶν γωνιῶν εὑρήσομεν τὸν ζητούμενον πολύγωνον.
> (“In fact, taking the side of the polygonal always doubling we shall subtract a unit, and multiplying the remainder by the ‹number› less by a dyad than the multiplicity of the angles we shall always add a dyad to the result, and taking the square on the result we shall subtract from it the ‹square› on the ‹number› less by a tetrad than the multiplicity of the angles, and dividing the remainder by the octuple of the ‹number› less by a dyad than the multiplicity of the angles we shall find the sought polygonal.”)

The algorithmic code resorts to paradigmatic examples featuring specific numerical values[45]. After the initializing clause, the prescriptions are expressed as a sequence of principal clauses coordinated by asyndeton; each clause formulates exactly one step of the algorithm and comprises a verb form in the imperative (this is the operation) and a system of two complements, a direct one and an indirect one, in the form of demonstratives or of numerical values (these are the operands). The operation is often expressed by means of the preposition introducing the indirect complement, without any verb form. The result of each operation is identified in a separated clause, with the verb in the present indicative (forms of γίνομαι “to yield”), sometimes replaced by an adjective in predicative position (mainly λοιπός “as a remainder” after a subtraction). Both syntactical structures are equivalent to our equals sign. The main feature of an algorithm is the systematic and exclusive use of parataxis by asyndeton: no coordinants, (almost) no connectors, no subordination. The algorithmic flow is usually one-step: any step 1) accepts as input a number that is directly the output of the immediately preceding step and 2) feeds in new data by means of the second operand. Operations in which neither operand comes from the immediately preceding step are less frequent. Such operations induce a hiatus in the algorithmic flow; the hiatus is often syntactically marked by the presence of particles or of

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[44] F. Acerbi (ed.), Diofanto, De polygonis numeris (*Mathematica Graeca Antiqua* 1). Pisa–Rome 2011, 197.18–30. Procedures prominently figure in the astronomical corpus; they expound how to use numerical tables to compute relevant astronomical quantities. Thus, we find procedures in Ptolemy, *Alm.* II.9, III.8, III.9, V.9, V.19, VI.9–10, XI.12, XIII.6, and the instruction manual to the *Handy Tables*, in Pappus' and Theon's commentaries thereon, in the anonymous *Prolegomena to the Almagest*, a late antiquity primer to the elementary arithmetical operations in the sexagesimal system, in Stephanus' commentary on the *Handy Tables*, and in all similar Byzantine primers. In the latter texts, procedures precede paradigmatic examples presented in algorithmic form and are intended to validate them.

[45] In the ancient Greek corpus, this code prominently figures in Hero's *Metrica*, and exclusively in the geometric metrological *corpus*. In the *Metrica*, proofs using the “language of the givens” precede paradigmatic examples of computations in algorithmic form, and are intended to validate them. In all astronomical primers mentioned in the previous footnote, paradigmatic examples presented in algorithmic form are very frequent; they are systematically preceded by procedures; as said, the latter are intended to validate the former. In these texts, algorithms are frequently replaced—or accompanied—by tabular arrangements of the performed operations; as a matter of fact, the latter are nothing but an evolution of the former in a more perspicuous format. In the computational primer included in Theodorus Meliteniotes' *Three Books on Astronomy*, each operation is described three times: by means of a procedure (called μέθοδος), of an algorithm (ὑπόδειγμα “example”), and of a tabular set-up (ἔκθεσις τῶν ἀριθμῶν “setting-out of the numbers”).

liminal verb forms. As an example of an algorithm we read a part of Hero, *Metr.* I.8—this is “Hero’s formula” for finding the area of a triangle once its sides are numerically given[46]:

| | |
|---|---|
| οἷον ἔστωσαν αἱ τοῦ τριγώνου πλευραὶ μονάδων ζ η θ. | For instance, let the sides of the triangle be of 7, 8, 9 units. |
| σύνθες τὰ ζ καὶ τὰ η καὶ τὰ θ· γίγνεται κδ· | Compose the 7 and the 8 and the 9: it yields 24; |
| τούτων λαβὲ τὸ ἥμισυ· γίγνεται ιβ· | take half of these: it yields 12; |
| ἄφελε τὰς ζ μονάδας· λοιπαὶ ε. | subtract the 7 units: 5 as a remainder. |
| πάλιν ἄφελε ἀπὸ τῶν ιβ τὰς η· λοιπαὶ δ. | Again, subtract the 8 from the 12: 4 as a remainder. |
| καὶ ἔτι τὰς θ· λοιπαὶ γ. | And further the 9: 3 as a remainder. |
| ποίησον τὰ ιβ ἐπὶ τὰ ε· γίγνονται ξ· | Make the 12 by the 5: they yield 60; |
| ταῦτα ἐπὶ τὰ δ· γίγνονται σμ· | these by the 4: they yield 240; |
| ταῦτα ἐπὶ τὰ γ· γίγνεται υκ· | these by the 3: it yields 720; |
| τούτων λαβὲ πλευράν, | take a side of these, |
| καὶ ἔσται τὸ ἐμβαδὸν τοῦ τριγώνου. | and it will be the area of the triangle. |

The algorithmic code employed in *Rechenbuch* problems is highly contaminated with procedures, and allows for several stylistic variations[47]. Some of them I shall explain in the commentary to the problems edited in this article.

## THE *RECHENBUCH* IN LAUR. PLUT. 86.3: *ANONYMUS* L

*Anonymus* L is contained in Laur. Plut. 86.3, a composite manuscript whose contents are as follows[48]: ff. 1r–162v Iamblichus, *Opera*[49]; ff. 163v–169v, material to be described below (2nd half of 13th century); ff. 171r–186v Marinus of Neapolis, *Vita Procli*, ff. 186v–204v [Aristotle], *De mirabilibus auscultationibus* (end 13th century + 16th-century restoration); ff. 205r–209v Theophrastus, *Characteres* (14th century); ff. 210r–232r Aeschylus, *Persae* (end 13th century). We are interested in the structure of the quinion ff. 161–170. It contains: ff. 161r–162v end of the collection of Iamblichus’ treatises; 163r blank; 163v–164r two divisions of the canon; 164v table of currency equivalence; 165r–169v *Anonymus* L; 170r–v blank. Since *Anonymus* L starts at f. 5 of the quinion, the *Rechenbuch* is, together with the other material, a filler intended to complete the Iamblichean transcription. There is only one hand at work in *Anonymus* L, despite the ink and pen change—entailing a slight variation of the ductus—at ff. 168v15–169v21.

Contrary to what is currently asserted[50], the hands involved in copying Iamblichus and *Anonymus* L must be definitively dated to the second half of the 13th century. In particular, the main copyist of

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46 Acerbi – Vitrac, Héron d’Alexandrie 174.3–7.

47 On the use of the first person singular in alloying problems, see again Høyrup, Fibonacci 236–238.

48 Descriptions in P. Moraux – D. Harlfinger – D. Reinsch – J. Wiesner (eds.), Aristoteles Graecus. Die griechischen Manuskripte des Aristoteles. Erster Band, Alexandrien–London. Berlin – New York 1976, 282–286 (by J. Wiesner); D. Saffrey – A.-Ph. Segonds – C. Luna (eds.), Marinus, Proclus ou sur le bonheur. Paris 2001, cvi–cix; C. Giacomelli, Un altro codice della biblioteca di Niceforo Gregora: il Laur. Plut. 86, 3 fonte degli estratti nel Pal. gr. 129. *Quaderni di storia* 80 (2014) 217–237: 219–222, and C. Giacomelli, Ps.-Aristotele, De mirabilibus auscultationibus. Indagini sulla storia della tradizione e ricezione del testo (*Commentaria in Aristotelem Graeca et Byzantina* 9). Berlin 2020. I also thank C. Giacomelli for discussions about the hands involved in ff. 1–170 of this manuscript. I also tacitly correct some datings of hands: F. Acerbi – A. Gioffreda, Manoscritti scientifici della prima età paleologa in scrittura arcaizzante. *Scripta* 12 (2019) 9–52.

49 These are ff. 2v–46v *De vita Pythagorica*, 47v–82v *Protrepticus*, 84r–115v *De communi mathematica scientia*, 115v–162v *In Nicomachi arithmeticam*.

50 The assertion is based on a misreading of N. G. Wilson, Nicaean and Paleologan Hands. Introduction to a Discussion, in: La paléographie grecque et byzantine. Actes du Colloque Paris, 21–25 octobre 1974, ed. J. Glénisson – J. Bompaire – J. Irigoin (*Colloques internationaux du C.N.R.S.* 559). Paris 1977, 263–267: 265, about the script of the first codicological unit of Laur.

*Anonymus* L is found in Vat. gr. 192, a manuscript also featuring the hand of the monk Ionas, who in its turn, subscribed Oxford, Bodleian Library, Roe 22 (Niketas Choniates) on 15 May 1286[51].

Let us now come to the mathematical material that precedes *Anonymus* L in Laur. Plut. 86.3. At ff. 163v–164r one finds two canonic divisions, the latter being a fairly incomplete redrawing of the former. This canonic division is a Greater Perfect System[52] that includes the names and standard signs of the notes, the ratios between consecutive notes, the main ratios between notes and the names of the corresponding musical intervals, and the numbers conventionally assigned to the notes. A marginal annotation counts how many times the main musical intervals figure in the diagram.

At f. 164v, the table transcribed just below lists the equivalence of a nomisma (the main currency in the Byzantine Empire) and of the fractional currency μιλιαρίσιον (12 μιλιαρίσια = 1 nomisma), and in addition, of the weight and fineness unit κεράτιον (24 κεράτια = 1 nomisma)[53]; the first and the last column indicate such equivalences assuming as the counting unit 1 (nomisma; left) and 6000 (right)[54]. Note the old names (albeit misspelled)[55] of the coins worth $\frac{1}{2}$ and $\frac{1}{3}$ of a nomisma.

| | | | |
|---|---|---|---|
| κδ$^{ον}$ | κεράτιν | κράτει | σν |
| ιβ$^{ον}$ | μιλιαρίσιον | κράτει | φ |
| η$^{ον}$ | γ κεράτια | κράτει | ψν |
| ϛ$^{ον}$ | β μιλιαρίσια | κράτει | ͵α |
| δ$^{ον}$ | γ μιλιαρίσια | κράτει | ͵αφ |
| γ$^{ον}$ | τριμίσιν | κράτει | ͵β |
| γ$^{ον}$ ιβ$^{ον}$ | ε μιλιαρίσια | κράτει | ͵βφ |
| 𐅵 | σιμίσιν | κράτει | ͵γ |
| 𐅵 ιβ$^{ον}$ | ζ μιλιαρίσια | κράτει | ͵γφ |
| 𐅵 ϛ$^{ον}$ | η μιλιαρίσια | κράτει | ͵δ |
| 𐅵 δ$^{ον}$ | θ μιλιαρίσια | κράτει | ͵δφ |
| 𐅵 γ$^{ον}$ | ι μιλιαρίσια | κράτει | ͵ε |
| 𐅵 γ$^{ον}$ ιβ$^{ον}$ | ια μιλιαρίσια | κράτει | ͵εφ |
| | τὸ ν$^{ο}$ ιβ μιλιαρίσια | κράτει | ͵ϛ |

## OVERVIEW OF THE MATHEMATICAL CONTENTS OF *ANONYMUS* L

I first provide information needed to understand what some problems in *Anonymus* L are about. This information consists in the basic equivalence rules among weights or currencies assumed as a matter of course in *Rechenbücher*. The rule for weights and the equivalence table of nominal values of currencies are as follows[56]:

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Plut. 86.3 "resembl[ing]" Triclinius'.

51 See Acerbi – Gioffreda, Manoscritti scientifici 16–24. A detailed analysis of the Bodleian manuscript is in A. Turyn, Dated Greek Manuscripts of the Thirteenth and Fourteenth Centuries in the Libraries of Great Britain (*DOS* 17). Washington DC 1980, 49–52 and pl. 28–31.

52 See A. Barker, The Science of Harmonics in Classical Greece. Cambridge 2007, 12–18.

53 See the following section for the complete equivalence table. Recall that the carat is not a currency (see again below).

54 For the basic monetary unit (here, the nomisma) being divided into 6000 parts, see Tannery, Le calcul; Morrisson, La *logarikè* 440–441; Baillet, Le papyrus mathématique; and D. H. Fowler, The Mathematics of Plato's Academy. Oxford 1999, 235–236 (papyri). On this choice see also probs. 13–18 and commentary thereon; the counting unit ranging as far as 6000 is the noummion. As a matter of fact, what is here set out in tabular form is an abridgment (with the addition of the carats entries) of the list opening the *Palaia Logarikê* in Par. gr. 1670, f. 3r–v. Edition of the list in N. G. Svoronos, Recherches sur le cadastre byzantin et la fiscalité aux XI$^{e}$ et XII$^{e}$ siècles: le cadastre de Thèbes. *BCH* 83 (1959) 1–145: 79; translation in tabular form in Hendy, Coinage 59, or Morrisson, La *logarikè* 422.

55 But for the several spellings of σιμίσιον see *LBG*, *sub voce*.

56 Cf. C. Morrisson, Byzantine Money: Its Production and Circulation, in: The Economic History of Byzantium. From the

| pound | ounce | exagion | gram | carat | nomisma | miliaresion | (carat) | follis |
|---|---|---|---|---|---|---|---|---|
| 1 | 12 | 72 | 288 | 1728 | 1 | 12 | 24 | 288 |
| | 1 | 6 | 24 | 144 | | 1 | 2 | 24 |
| | | 1 | 4 | 24 | | | 1 | 12 |
| | | | 1 | 6 | | | | |

Information about other weight or currency units is provided in the commentary *ad loca*. The reader is also referred to the word index below, to E. Schilbach's books on Byzantine metrology[57], and to the indexes of edited *Rechenbücher*[58].

*Anonymus* L contains 48 problems. They can be categorized within two different typologies, on the basis either of their "bare" mathematical content or of their staging format. A non-exclusive mathematical typology is as follows[59].

- calculation of interest: 13–18;
- calculations with fractions, both unit and common fractions: 32–38;
- Diophantine-style problems in everyday-life guise: 1, 2 (telling the hour: an unknown number plus a part of itself yields a given number: no counterpart in Diophantus' *Arithmetica* since it involves one variable only; cf. *AP* XIV.6, 139–142); 7 (give-take problem: assigned exchange-fractions and equal, and assigned, final amount: Diophantus, *Ar*. I.21); 8, 10, 11 (give-take problems: assigned exchange-amount and assigned final ratios [one of them ratio of equality]: *Ar*. I.15; *AP* XIV.145–146); 26 (cup made of two metals: system of two equations in two unknowns: *Ar*. I.5; cf. *AP* XIV.13); 39, 43, 44 (pursuit: an unknown number plus a given number is equal to a suitable rescaling of the unknown number);
- iterative partitions: 40, 45 (apples, beggars);
- proportional partition of a given amount (always tripartition; bipartition in *Ar*. I.2; frequent in *AP* XIV): 4 (tank filled by three sources; cf. *AP* XIV.7); 5, 6 (estate partitioned among three people), 12 (generic purchase), 41 (purchase of a drink by three people);
- multiplication by several numbers: 3 (telling the hour);
- rule of three: 19–24 (values of alloy with variable fineness); 25 (conversion of units of measurement: weights and currencies); 27 and 28–31 (conversion of units of measurement; 31 gives a rule); 34–36 (change of denomination of fractions); 42 (bees eating honey); 46 (bow killing birds); 47, 48 (buying goods; entails conversion of units of measurement);
- onomatomancy: 9.

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Seventh through the Fifteenth Century, ed. A. E. Laiou (*DOS* 39). Washington DC 2002, 891–966: 921 and 930, Hendy, Coinage 25. Recall that the weight of a nomisma is 1 exagion = 24 carats; this means that a standard gold nomisma is of 24 carats weight and of 24 carats fine. The carat was thus also used as the fineness unit (that is, a unit of value), but it was not a currency. The miliaresion and the follis were originally a silver and a copper coin, respectively; after Alexios I's reform, they became units of account not represented by a coin. The miliaresion loses even this function from the mid 12th-century on, and in fact it is never mentioned in our *Rechenbücher*. A clear exposition of Byzantine monetary terminology is in Hendy, Coinage 27–38. See also C. Morrisson, Les traités d'arithmétique byzantins des XIIIe–XVe siècles, source d'histoire monétaire. *Revue Numismatique* 167 (2011) 171–183, for a short discussion of the currencies mentioned in the *Rechenbücher* edited so far.

57 E. Schilbach, Byzantinische Metrologie (*HdA* 12.4). Munich 1970, and Schilbach, Byzantinische metrologische Quellen, for the sources.

58 See Tannery, Notice 188–198 (Rhabdas' explanations in his *Letter to Tzavoukhes* are invaluable); Vogel, Ein byzantinisches Rechenbuch 139–145 and 161–163; Deschauer, Die große Arithmetik 359–413.

59 Compare the analogous typologies in Vogel, Ein byzantinisches Rechenbuch 147–148; Hunger – Vogel, Ein byzantinisches Rechenbuch 87–91; Chalkou, The Mathematical Content 28–56; Deschauer, Die große Arithmetik 355–357.

A non-exclusive typology based on the staging format and everyday-life goals is instead as follows (details on the actual staging in the previous typology):

- alloy currencies: 19–24;
- alloying: 26;
- conversion of units of measurement: 19–31, 47, 48;
- interest rates: 13–18;
- give-take: 7, 8, 10, 11;
- handling fractions: 32–38;
- onomatomancy: 9;
- lively staging: 1–6, 40–42, 45, 46;
- pursuit: 39, 43, 44;
- sellying-buying: 12, 41, 47, 48;
- telling the hour: 1–3.

The following table sets out the structure of *Anonymus* L according to the previous typology; the second and the fourth row contain the concordance with *Anonymus* P:

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 62 | 62 | / | 64 | 71 | / | 72 | / | / | 73 | / | / | / | / | / | / | / | / | 74 | / | 75 | 76 | 77 | 78 |
| 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 |
| 79 | / | / | / | / | / | 80 | 81 | / | 82 | / | 83 | / | / | 24 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | / | / |

## RESOLUTIONS OF COMMON FRACTIONS INTO UNIT FRACTIONS CONTAINED IN *ANONYMUS* L

Dealing with common fractions by resolving them into unit fractions was a current technique in the Greek and Byzantine world[60], and more generally within the Mediterranean basin. Systematic lists of resolutions into unit fractions are found in the manuscript tradition and in papyri[61]. A complete table is in the Papyrus Achmin: denominations from 2 to 20, including $\frac{2}{3}$; the numerators are units, tens, hundreds, thousands, and 1 myriad as far as the denomination 10, if instead the denominations fall in the range $11 \le n \le 20$, the numerators go from 1 to $n$; only one resolution is set out[62]. The list of resolutions of common fractions in *Anonymus* 1183, Par. gr. 1670, ff. 44v–46v, is transcribed and translated in the Appendix[63]. Simpler resolution tables are attached to Rhabdas' *Letter to Khatzykes*[64];

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[60] On this issue, see W. R. Knorr, Techniques of Fractions in Ancient Egypt and Greece. *Historia Mathematica* 9 (1982) 133–171; B. Vitrac, Logistique et fractions dans le monde hellénistique, in: Histoire de fractions, fractions d'histoire, ed. P. Benoit – K. Chemla – J. Ritter. Basel 1992, 149–172; Acerbi – Vitrac, Héron d'Alexandrie 81–84 (Hero's *Metrica*).

[61] List of this kind of tables in papyri in Fowler, The Mathematics 269–274; edition of one of them in F. E. Robbins, A Greco-Egyptian Mathematical Papyrus. *Classical Philology* 18 (1923) 328–333.

[62] Similar tables, going as far as the ninths, are found in Vat. gr. 1058, ff. 36v–38r (early 15th century).

[63] Parts expressed as sums of unit fractions are systematically used in the *Palaia Logarikê* and *Nea Logarikê* in the same manuscript.

[64] The tables are edited in Tannery, Notice 114–117; in the manuscripts, see Vat. gr. 1411, f. 13r, Venezia, Biblioteca Nazionale Marciana, gr. Z. 323 (coll. 639), ff. 35v–36r (same copyist as Vat. gr. 1058). Rhabdas' *Letter to Khatzykes* is not a *Rechenbuch* but a computational primer; it contains the following (references are to the pages of Tannery, Notice): denominations of numbers and how to represent integers from 1 to 9,999 on the fingers of the hands (86.1–96.12); abstract descriptions of the five elementary arithmetic operations on integers, extraction of an approximate square root included (96.13–102.9); denominations of numerical orders and their multiplication (102.10–110.5). A structured set of tables of addition, subtraction,

they were almost certainly contained in the anonymous treatise that Rhabdas plagiarized[65]. The following tables set out all resolutions of common fractions into unit fractions used in *Anonymus* L.

Fifths

| | |
|---|---|
| numerator | 4 |
| resolution | $\frac{1}{2}\ \frac{1}{5}\ \frac{1}{10}$ |
| problem | 23, 24, 28 |

Sevenths

| | | | | |
|---|---|---|---|---|
| numerator | 3 | 4 | 5 | 6 |
| resolution | $\frac{1}{6}\ \frac{1}{7}\ \frac{1}{14}\ \frac{1}{21}$ | $\frac{1}{2}\ \frac{1}{14}$ | $\frac{1}{2}\ \frac{1}{7}\ \frac{1}{14}$ | $\frac{2}{3}\ \frac{1}{7}\ \frac{1}{21}$ |
| problem | 1, 19, 36, 42 | 19, 34 | 47 | 36 |

Eights

| | | |
|---|---|---|
| numerator | 5 | 7 |
| resolution | $\frac{1}{2}\ \frac{1}{8}$ | $\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$ |
| problem | 45 | 45 |

Ninths

| | |
|---|---|
| numerator | 2 |
| resolution | $\frac{1}{6}\ \frac{1}{18}$ |
| problem | 2 |

Tenths

| | |
|---|---|
| numerator | 7 |
| resolution | $\frac{1}{2}\ \frac{1}{5}$ |
| problem | 30 |

multiplication, and partition is found at the end of the treatise and was apparently meant to complete it; it also contains an introduction to the partition table (114.1–17).

[65] See F. Acerbi – D. Manolova – I. Pérez Martín, The Source of Nicholas Rhabdas' *Letter to Khatzykes*: An Anonymous Arithmetical Treatise in Vat. Barb. gr. 4. *JÖB* 68 (2018) 1–37.

## Eleventhths

| numerator | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| resolution | $1/12\ 1/22\ 1/33\ 1/44$ | $1/9\ 1/18\ 1/99\ 1/198$ | $1/4\ 1/44$ | $1/3\ 1/33$ | $1/3\ 1/11\ 1/33$ | $1/2\ 1/22$ | $1/2\ 1/11\ 1/22$ | $1/3\ 1/4\ 1/11\ 1/33\ 1/44$ | $1/2\ 1/6\ 1/22\ 1/66$ | $2/3\ 1/22\ 1/66$ | $1/2\ 1/4\ 1/22\ 1/44$ |
| problem | 2, 4, 38, 44 | 2 | 4, 26 | 26, 38 | 12, 38 | 4 | 26 | 12, 26, 38 | 26 | 38 | 2, 12 |

## Twelfths

| numerator | 5 |
|---|---|
| resolution | $1/4\ 1/6$ |
| problem | 5 |

## Thirteenths

| numerator | 4 | 8 |
|---|---|---|
| resolution | $1/6\ 1/13\ 1/26\ 1/39$ | $1/2\ 1/13\ 1/26$ |
| problem | 46 | 35 |

## Other fractions

| fraction | $1/2$ | $1/3$ | $1/4$ | $3/17$ | $9/19$ | $11/24$ | $12/25$ | $2/33$ | $13/33$ |
|---|---|---|---|---|---|---|---|---|---|
| resolution | $1/4\ 1/6\ 1/12$ | $1/5\ 1/12\ 1/20$ | $1/5\ 1/20$ | $1/12\ 1/17\ 1/51\ 1/68$ | $1/4\ 1/6\ 1/38\ 1/57\ 1/76$ | $1/3\ 1/8$ | $1/5\ 1/6\ 1/10\ 1/75$ | $1/22\ 1/66$ | $1/3\ 1/22\ 1/66$ |
| problem | 32 | 6 | 6 | 32 | 32 | 25 | 29 | 26 | 38 |

| fraction | $9/47$ | $42/47$ | $43/47$ | $47/60$ | $24/125$ | $127/250$ | $2/3 - 1/11 - 1/17 = 96\frac{2}{3}/187$ |
|---|---|---|---|---|---|---|---|
| resolution | $1/6\ 1/47\ 1/282$ | $1/2\ 1/4\ 1/8\ 1/94\ 1/188\ 1/376$ | $1/2\ 1/3\ 1/15\ 1/94\ 1/235$ | $1/3\ 1/4\ 1/5$ | $1/6\ 1/75\ 1/125\ 1/250$ | $1/2\ 1/125$ | $1/5\ 1/6\ 1/11\ 1/170\ 1/187$ |
| problem | 5 | 5 | 5 | 5 | 29 | 29 | 37 |

| fraction | $210/323 = 3/17 + 9/19$ | |
|---|---|---|
| resolution | $1/2\ 1/10\ 1/20\ 1/6460$ | $1/2\ 1/17\ 1/38\ 1/51\ 1/57\ 1/68\ 1/76$ |
| problem | 32 | 32, 33 |

## THE SET OF PROBLEMS IN VAT. GR. 191, F. 261R: *ANONYMUS* J

Vat. gr. 191 is a late 13th-century manuscript in oriental paper; it is written by sixteen copyists, named hands A to Q in recent scholarship, and it comprises several thematic and codicological blocks[66]. Vat. gr. 191 is commonly (and wrongly) held to be a paradigmatic instance of a codex assembled by cooperating copyists coordinated by a supervisor[67]. Within the block made of the astrological collection at ff. 229–286 (elsewhere penned by hand K alone), a page written by hand J is found: it is f.

[66] See Turyn, Codices graeci Vaticani 89–97; D. Bianconi, Libri e mani. Sulla formazione di alcune miscellanee dell'età dei Paleologi. *Segno e Testo* 2 (2004) 311–363: 324–330 and fig. 1; Acerbi – Gioffreda, Manoscritti scientifici 41–44.

[67] For arguments against the standard view, see F. Acerbi, Byzantine Recensions of Greek Mathematical and Astronomical Texts: A Survey. *Estudios bizantinos* 4 (2016) 133–213: 192–195, and Acerbi – Gioffreda, Manoscritti scientifici 30–34.

261r, where the text is deleted by two long, crossed pen strokes. The beginning of the text at f. 261v exactly fits the end of that at f. 260v; the text at f. 261r is a portion of a *Rechenbuch* and has nothing to do with the text surrounding it, nor with anything elsewhere in Vat. gr. 191: thus, the presence of hand J here, which however copied other parts of the manuscript, is just a matter of recycling paper. This micro-*Rechenbuch* contains six problems; the last item ends exactly at the end of the page and the verso of the folio was originally blank: the collection might well be complete. The typology is as follows:

- Give-take problems: a, b, d.
- Casting lots by dice: c.
- The riddle of the ring: e.
- Sum of an arithmetic progression: f.

Here is the concordance table with *Anonymi* L, P, 1306, and V:

| J | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| L | 8, 10, 11 | 8, 10, 11 | / | 8, 10, 11 | / | / |
| P | / | / | 100 | / | / | 111–112 |
| 1306 | III.1[68] | / | / | / | / | / |
| V | / | / | / | / | 38 | / |

The six problems in *Anonymus* J do not use unit fractions and display, as for particles and adverbs, a slightly different lexicon from that of *Anonymus* L: reading the texts and going through the word index in the next section will make this characteristic apparent. With respect to *Anonymus* L, noteworthy features are the more frequent use of connexive λοιπόν and the exclusive presence of the adverbs αὖθις and πάντοτε, the compartmented lexicon for subtraction (κουφίζω L vs. ὑφαιρέω J, the latter with geminated *lambda* in aorist tense forms, and the term ὑφειλμός), the use of τόσα for the unknown (sounding *so* similar to Italian *cosa* and never used in other *Rechenbücher*, but it may be sheer coincidence)[69], and the participle ἐκφωνούμενον for an assigned number. The style of *Anonymus* J is more discursive, less rigidly algorithmic, eager to spell out general rules.

## THEMATIC WORD INDEX OF *ANONYMI* L AND J

This word index is also intended as a glossary to the translation; I have tried to follow the principle of translating different terms in Greek with different terms in English, even if the rich preverbal system ancient Greek avails of sometimes makes it impossible to establish a one-to-one correspondence—and even if the outcome is at times bizarre[70]. It might have sounded bizarre to such ancient Greek ears as Hero's, too, for the *Metrica* displays a remarkable lexical uniformity in this respect[71]. The wide, and sometimes slightly bewildering, range of prepositions used to mark the second operand of an operation coincides with that of *Anonymus* P[72]. Forms in restored clauses are marked by an asterisk. The problems in *Anonymus* L are numbered from 1 to 48, those in *Anonymus* J from a to f.

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68 This is item 1 of the section of *Anonymus* 1306 I have called above μέθοδοι καθολικαί.

69 I thank J. Høyrup for a discussion on this point. The term *cosa* for the unknown does not seem to be used before Jacopo da Firenze's *Tractatus algorismi* of 1307. Note, however the use of τόσσα in *AP* XIV.144.

70 There are also some English neologisms; see the following section.

71 See Acerbi – Vitrac, Héron d'Alexandrie 74–81.

72 See Vogel, Ein byzantinisches Rechenbuch 141–143, and compare with the discussion mentioned in the previous footnote.

### *Non-lexical items*

***Numerical entities***. ἀριθμός: number (3–5, 44, b–e); δεκάς: decad (e); ἑπτάς: heptad (c); μαλλίον: mallion (38); μέρος: part (4–6, 44, 46); μονάς: unit (13–17, 32, 38); οὐδέν: nothing (45); πισθομόριον: further part (38); στερεός: solid ‹number› (38); φωνή; denomination (5, 32, 34, 38, 44); χιλιάς: thousand (e); ψηφίον: counting-unit (28, 31); ψῆφος: part (38).

***Unknown quantities***. ὅσος: what, how much, *as much* (6, 36, 39, *c*, *e*); ποσόν: quantity (31); πόσος: what, how much (4, 4, 7, 8, 10–12, 16, 21–23, 25, 27–29, 34–36, 39, 40, 42–44, 46, 48, f); τοιοῦτος: such (36); τόσος: such-and-such, such (b, f); τοσοῦτος: such (6).

***Operations***

*Addition*. ἐπαναλαμβάνω ἐπί: to take up in addition on (39); μίγνυμι: to merge (38); ὁμαδεύω: to collect (38); ὁμάς: collection (3); ποιέω followed by a conjunction: to do (12, 37); προστίθημι εἰς, ἐπί: to add to (1, 2, 7, a, a*, c, e, f); τίθημι εἰς: to set to (26).

*Subtraction*. αἴρω ἐκ: to raise from (d, e); ἀφαιρέω: to remove (f); ἐκβάλλω: to take away from (a); ἐπαίρω: to raise (40); κουφίζω ἐκ, εἰς: to subtract from, to (7, 19, 26, 37, 43, a*); ὑφαιρέω ἐκ, ἀπό: to remove from (37, c, e); ὑφειλμός: removal (c).

*Multiplication*. ἀναλαμβάνω εἰς: to take up on (9); ἐπαναβαίνω εἰς: to mount on (5); ἐπιβάλλω: to put upon (31); ποιέω ἐπί, εἰς: to do by, into (1, 2, 4, 10, 12, 13, 19, 20, 25, 26, 28–30, 33, 33, 36, 42–44, 46)[73]; πολλαπλασιάζω ἐπί, εἰς: to multiply by, into (14, 15, 17, 18, b, f); πολλαπλασιασμός: multiplication (31, b); πολυπλασιάζω ἐπί: to multiply by (33).

*Multiples*. ἅπαξ: once (19, 20); δεκαπλασιάζω: to decuplicate (c, e); δεκαπλασιασμός: decuplication (c); δεκαπλόω: to decuplicate (3); διπλάζω: to double (c, e); διπλάσιος: the double (d); διπλός: the double (b); διπλόος: twofold (8); διπλόω: to double (3, 25, 45); δωδεκαπλασιάζω: to dodecuplicate (d); εἰκοσαπλόω: to twentuplicate (32); ἐνναπλασιάζω: to ennuplicate (c); ἐνναπλασιασμός: ennuplication (c); ἑξαπλόω: to sextuplicate (40); πενταπλασιάζω: to quintuplicate (c, d, e); πενταπλόω: to quintuplicate (c, d, e); τετραπλόος: fourfold (11); τετραπλόω: to quadruplicate (41); τριπλασιασμός: triplication (c); τριπλόος: threefold (10); τριπλόω: to triplicate (3, 41, c).

*Submultiples*. δίμοιρον: two-thirds (37); ἥμισυ(ν), ἥμισο: a half (12, 12, a); ποιέω + genitive: to do of (5, 16, 44, 45).

*Division*. ἀναλύω εἰς: to resolve out into (3, 24, 28, 31); ἀπολύω: to resolve off (4, 5); ἐπιλύω: to resolve (34); λύσις: resolution (38); λύω εἰς: to resolve into (2, 3, 13–15, 17, 18, 21, 25, 29, 34–36, 38, 46–48); μερίζω εἰς: to divide into (31, 37, b); ποιέω εἰς: to do into (25, 33); συγκρίνω πρός: to compare to (32).

*Result*. ἀπομένω: to remain (e); γίνομαι: to yield (1–2, 4–7, 9, 12–26, 28–30, 32–48, a, c, e); (κατα)λείπω: to leave (out) (37, c, c); καταλιμπάνω: to leave out (c, d, e); λοιπός: as a remainder (7, 19, 20, 26, 40, 43, 45); μένω: to remain (37, 40, 45); ὅλος: whole (3, 26, 46, c, e); ὁμοῦ: together (1, 2, 4, 5, 12, 20, 25, 26, 28–30, 32, 33, 38, 40, 41, 44–46, c, f); περιττεύω ἀπό: to remain over from (c); ποιέω: to make (3, 13–17, 25, 26, 32, 34–36); (συν)ἀθροίζω: to put together (31, 38); συνάγω: to gather (2, 3, 5, 12, 26, 31, 32, 38); ὑπολείπω: to leave over (45); ὑπόλοιπος: left over (45).

*Proportionality*. ἀναλόγως: in proportion (12, 41).

*Factoring out*. γυρεύω: circumvent (e).

***Alloys***. ἀργυρός: silver (25, 26); κεράτιον: carat (5, 20–29, 47, 48); χρυσίος ἀργυρός: white gold (21–23); χρυσίος: gold (26); χρυσὸς ἀργ(υρ)ός: white gold (19, 20, 22, 23).

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Recall that in this kind of texts an operation is frequently identified by the sole preposition. Multiplication may even be formulated by mere juxtaposition of the factors, as in our probs. 1–4, 8–10, 18, 28, 30, 32–39, 39, 46, a, d, f. Probs. 19 and 20 have the phrase ποιοῦμεν ἅπαξ.

73 Very frequently without a preposition, see the previous footnote.

***Currencies***. ἡμίσιον: semissis (12); καθαρός: pure (29); νόμισμα: nomisma (5–8, 10–20, 25–30, 47, 48); νουμμίον: noummion (12, 45); τραχίον: trachion (41); τριμίσιον, τρίμισυν: tremissis (12); χάραγμα νόμισμα: coined nomisma (19).

***Interest***. δανείζω ὑπὲρ/ὑπὸ τόκων: to lend at interest (13, 14, 15, 16); δίδωμι: to give (15–18); λαμβάνω: to take (13–16); τελεία ἑκατοστή: full per cent rate (18); τόκος: interest (13–18).

***Lengths***. μήλιον, μίλιον: mile (43); στάδιον: stadium (39).

***Pursuit***. εἰσέρχομαι: to come into (39); πήδημα: leap (44); προεξέρχομαι: to set out before (39); προκόπτω: to be in advance (44); προλαμβάνω: to be ahead (39, 43, 44); φθάνω: to overtake (39, 43, 44).

***Selling and buying***. ἀγοράζω: to buy (12); ἀκρόλιον: first-fruit (27); βαστάζω: to hold (7, 8, 10, 45, a, b, d, f); δίδωμι: to give (40, 41, 45, 47); δίδωμι: to give (7, 8, 10, 11, a, b, d) in give-take; ἐπαίρω: to raise (22); ἐπιδίδωμι: to give (b) in give-take; ἔρχομαι: to amount to (25, 27); λαμβάνω: to take (19, 20, 24, 40, 45, 48, f); λαμβάνω: to take (6, 8, 10, a, b) in give-take; μαργαρίτα: pearl (28); παρέχω: to provide (45); πιπράσκω: to sell (47); προτείνω: to offer (10) in give-take; τιμή: value (21, 24–26, 28–30); τίμημα: value (31).

***Weights***. γραμμόν: gram (25–27); ἐξάγιον: exagion (12, 19–24); κεράτιον: carat (19, 25, 26, 28–30); κερατισμός: carat-value (31); κοκκί(υ)ον: grain (28, 29, 30, 31), λίτρα: pound (25, 26, 41, 42); μόδιον: modius (47, 48); οὐγγία: ounce (25–30, 32, 42); οὐγγιασμός: ounce-value: (31); στατήρ: stater (28); στένω: to balance (28–30).

*Lexical items*

***Connectors, particles, and adverbs***. ἀλλά: but (8, 10, 11, f); ἀπό: each (7, 8, 10, 26); ἄρτι: now (45, c); αὖθις: anew (e); γάρ: in fact, for (25, 32, 46, d); εἶτα: afterwards (3, b, d); ἐπεί: since (42); ἐπειδή: because, since (1, 2, 2, 12, 16, 18, 19, 25, 40, 44); ἵνα: in order that (1, 2, 27); καί: also, too (3–6, 27, 28, 34, 38, 40, 41, 45, a, c); λοιπόν: finally (3, a, d, e); ὁμοίως: similarly (3, 5, 6, 40, 45, a, f); ὅτι: that, as, *because* (6, 8, 10, 11, 28, 38–40, 41–43, *44*, 46, *a*, c, f); οὖν: then (1–8, 10, 12, 16–28, 30, 32, 34–36, 38, 40, 42–46, 48, a, c–e); οὐχί: not at all (8, 10, 11); πάλιν: again (3, 5, 6, 37, 38, b–e); πρότερον: before (45); ὡς: so that, *so as to*, as (1, 3, 4, 8, 10, 19, *28*, *29*, *38*, 39, 40, 42, 45).

***Generality***. ἀεί: always (b, c); πάντοτε: always (b, e).

***Initialization and winding up***. ἀποτίθημι: to keep away (20); κρατέω: to keep (5, 8, 10, 11, 19, 38, 40, b, c, e); λαμβάνω: to take (7, 20); λέγω: to say (1, 3, 4, 5, 7, 8, 10–13, 16, 26, 39–43, 45, 467, a, d–f); ποιέω: to do (8, 9, 11, 26, 32, 34, 37, 39, 41, 42, 47, f).

***Metadiscursive***: ἀναμάρτητος: faultless (6); ἀνωτέρω: above (11); ἀπαιτέω: to ask (3); βραχύς: short (34); γιγνώσκω: to be aware of, to know (4, f); δῆλον: clear (38, 40); δηλονότι: clearly (3, 4, 27, 39, 42); διὰ τί: why (1, 2, 25, 42, 44, e); διὰ τό + noun or infinitive: because (of) (25, 42, 44, 48, a); διότι: because (e); διπλῶς: in two ways (27); εἶδον: to see, to know (26, 38, e); ἐκφωνέω: to utter (d); ἐπερωτάω: to ask (1, 3); ἐπιδείκνυμι: to show (34); ἐρωτάω: to ask (3, a, b, d); ἐρώτημα: question (a); ἐρώτησις: question (2–4, 10, 11, 14–17, 20–22, 24, 26, 29, 30, 48, f); εὑρίσκω: to find (3–5, 25, 31, 38); εὐχερῶς: easily (31, 38); ἰδού: there it is (7, 8, 10, 34, 40); ἰστέον: one has to know that (6, 28, 38); ἵστημι: to stand (6, 26); κανών: rule (3, 11, 25, 38); μέθοδος: procedure (1–5, 8, 10, 12–16, 19, 21–29, 32, 33, 34, 36, 38, 40, 42, 43, 45, 46, a, b, d, f); νοέω: to conceive (25, 27, c); οἷον: for instance (38); οὕτω: in this way, thus (9, 36); οὕτως: as follows, so (3, 8, 9, 26, 32, 34, 37–39, 41, 42, 44, 44, 45, 45, 47, d, f); συμβάλλω: to occur (31); συνίστημι: to conjure up (32); τουτέστι: that is (19, 28, 31, 38); ὑποδείξεως χάριν: for the sake of (3, 27, 28); φέρω: to convert (9, 31); φιλομαθέστατε: you fondest of learning (38); φιλοπόνως: industriously (38); ψῆφος: calculation (1, 5, 6, 13, 19, 25, 27, 28, 31).

***Modalities and imperativals***. εἰπεῖν: say (5, 26, 41, 42); χρή: one must (4–7, 40); ὠφείλω: ought to (6, 13–15, 38, 48, c–e).

***Particulars***. μέλλω + infinitive translated with conditional (46); προσέθηκε (perfect tense): turns out to add (1, 2); ὡς πρός strengthened preposition (28, 30).

***Pronouns***. ἀμφότερος: both of them (7, c); ἐκεῖνος: that, that guy (3, 31); –περ: indeed (3, 5, 27, 38, 43, 45); οὗτος: this (3, 4, 5, 12, 22, 23, 24, 25, 28, 29, 30, 32, 38, 40, 41, 45, 47, b, f); τι: something, what (3, 5, 13–20, 24, 26, 32, 37, 41, 45, 47); τις: someone, some (1, 3–5, 7, 8, 10, 12, 13, 16, 39, 43, 45, 47, a, b).

## PRELIMINARIES TO THE EDITION

The Greek text is generally edited as it stands, the exceptions mainly concern numerals; the expected reading is given in the apparatus; forms that are aberrant in classical Greek are kept in the main text. I have rigidly conformed to the conventions of the manuscript as for the accent of enclitics and as for the presence of movable *ny* and *sigma*. Deletions are included in square brackets and are usually not translated; restitutions—which include some rubricated initial letters—are between angular brackets and are translated. If the text has a lacuna that cannot be supplied with reasonable certainty, I have refrained from doing this, while explaining the issue in the commentary associated with the problem. I have transcribed the Greek numeral letters representing cardinals as simple letters, those representing ordinals (that is, the denominations of fractions) by putting a desinency at the exponent of the numeral letter, thus: $\gamma^{\text{ον}}$ “a third”; no apices are introduced. When the denomination is indicated in the text by doubling the numeral, I have written $\gamma\gamma^{\alpha}$ “thirds”. The fractions $\frac{1}{2}$ and $\frac{1}{3}$ are noted ∠ and ω, respectively.

My edition normalizes the punctuation: in a technical treatise, there is really no point in adhering to Byzantine conventions in such matters. Within the procedure or the proof of a problem, consecutive steps of the algorithm are separated by an upper point; a hiatus is marked by a full stop; commas are only introduced when ambiguities might arise, and sometimes to separate the result of a multiplication from the two factors[74]. The title system of *Anonymus* L, always penned by the main hand, is usually located in the margins of the manuscript page; I shall not indicate this feature in my apparatus, but enclose such titles in brackets with the indication “marg.”. *Anonymus* L also carefully marks the articulation enunciation-procedure-proof in each problem by means of rubricated, majuscule initials.

The reader will forgive me for the weirdness and artificiality of my translation. For uniformity’s sake, I have coined such words as “to twentuplicate”; by contrast, some terms are simply transliterated. Integrations occurring only in the translation are enclosed by smaller angular brackets. The procedure and the proof are punctuated as follows: a colon preceds the statement of a result; a semicolon separates steps in which the output-input chain is not interrupted; a full stop indicates an algorithmic hiatus and precedes the final winding up, where the solution is identified as such.

In the commentary, I have provided specific mathematical information about each problem, as well as an algebraic transcription of the procedure adopted, under the headings *Equation* and *Algorithm*. The latter is intended to represent faithfully the algorithmic flow of the procedure: steps in which the output-input chain is not interrupted are linked by an arrow; the operands in a given step are written in the same order as that in which they are introduced in the text[75]; the sign | separates independent results within one and the same step (that is, a branching has occurred); a full stop indicates an algorithmic hiatus. This symbolic transcription tends to eliminate the result of each operation, but I was

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[74] These are a part of the recommendations in ACERBI – VITRAC, Héron d’Alexandrie 98.

[75] If two consecutive steps formulate the same operation, the algorithm only reproduces the first.

unable to do better. Both *Equation* and *Algorithm* generalize, by introducing schematic letters, the paradigmatic example contained in the text. To see how my algorithmic transcription works, take for example prob. 1, where one reads "*Equation*. $x + (a/b)x = k$, with $(a,b,k) = (1,7,12)$. *Algorithm*. $(a,b,k) \rightarrow bk \rightarrow [1/(b + a)]bk = x$". This means that the intended equation is $x + (1/7)x = 12$ and that the algorithm is $7\times12 = 84$; $[1/(7 + 1)]\times84 = 10\ 1/2$. Commentaries on a string of similar problems are usually provided on the occurrence of the first of them.

Each problem is numbered. After the number I have indicated within brackets problems in other *Rechenbücher* that appear to be (nearly) identical to the one at issue; the absence of any such problem is denoted by three asterisks ***. I refrained from listing sets of similar but not identical problems in other *Rechenbücher*, for they can be found immediately by means of the typologies mentioned in note 59 above. I have instead systematically provided references to such problems in the Papyrus Achmin and in *AP* XIV.

## EDITION, TRANSLATION, AND COMMENTARY OF *ANONYMUS* L

Laur. Plut. 86.3, ff. 165r–169v

### 1

[= *Anonymus* P, no. 62 = *Anonymus* 1306, item 1 of ψηφιφορικὰ προβλήματα πάνυ ὀφέλημα]
|[165r] ψῆφος τῶν ὡρῶν.

τίς τινὰ ἐπερωτᾷ· ποία ἐστὶν ὥρα; λέγει· τῶν παρελθουσῶν ὡρῶν πρόσθες τὸ ζ$^{ον}$, ἵνα πληρωθῇ ἡ ἡμέρα, καὶ αὕτη ἐστὶν ἡ ὥρα.

μέθοδος. Ἐπειδὴ ζ$^{ον}$ προσέθηκε, ποίησον ζ ιβ· πδ· (διὰ τί δὲ ἐπὶ δώδεκα; ἐπειδὴ ἡ ἡμέρα ιβ ἐστὶν ὡρῶν·) τὸ η$^{ον}$ τῶν πδ· (διὰ τί δὲ τὸ η$^{ον}$; ἐπειδὴ ζ$^{ον}$ προσέθηκεν, ὅ ἐστιν η ζζ$^{α}$·) γίνεται οὖν τὸ η$^{ον}$ τῶν πδ, ι 𐅵. ἔστιν οὖν ι 𐅵 ὥρα· πρόσθες τὸ ζ$^{ον}$ τῶν ι 𐅵· γίνεται α[76] 𐅵· ὁμοῦ γίνονται ιβ.

Ἀπόδειξις. τὸ ζ$^{ον}$ τῶν ι· γίνεται α ϛ$^{ον}$ ζ$^{ον}$ ιδ$^{ον}$ κα$^{ον}$. καὶ τὸ ζ$^{ον}$ τοῦ 𐅵· γίνεται ιδ$^{ον}$[77]· ὁμοῦ α 𐅵· καὶ ι 𐅵· γίνονται ιβ. ἔστιν οὖν, ὡς εἴπομεν, ὥρα ι 𐅵.

Calculation of hours.

Someone asks someone: what time is it? He says: add $^1/_7$ of the past hours in order that the day be completed, and this is the time it is.

Procedure. Since he turns out to add $^1/_7$, do 7 ‹by› 12: 84; (and why by twelve? Because a day is of 12 hours;) $^1/_8$ of 84; (and why $^1/_8$? Because he turns out to add $^1/_7$, which ‹yielding› is $^8/_7$:) then $^1/_8$ of 84 yields 10 $^1/_2$. Then it is 10 $^1/_2$ o'clock; add $^1/_7$ of 10 $^1/_2$: it yields 1 $^1/_2$: together they yield 12.

Proof. $^1/_7$ of 10: it yields 1 $^1/_6$ $^1/_7$ $^1/_{14}$ $^1/_{21}$. And $^1/_7$ of $^1/_2$: it yields $^1/_{14}$: together 1 $^1/_2$; and 10 $^1/_2$: they yield 12. Then it is, as we have said, 10 $^1/_2$ o'clock.

**Problems 1–2**. An unknown number plus a part of itself yields an assigned number. The setting of telling the hour is a classical one: cf. *AP* XIV.6, 139–142. In both problems, the procedure is followed by two computational checks that the found number actually solves the problem; the second is more detailed than the first. *Equation*. $x + (a/b)x = k$, with $(a,b,k) = (1,7,12)$. *Algorithm*. $(a,b,k) \rightarrow bk \rightarrow [1/(b + a)]bk = x$.

### 2

[= *Anonymus* P, no. 63 = *Anonymus* 1306, item 2 of ψηφιφορικὰ προβλήματα πάνυ ὀφέλημα]
Ἄλλη[78] ἐρώτησις.

Τῶν παρελθουσῶν ὡρῶν πρόσθες ϛ$^{ον}$ ιη$^{ον}$, ἵνα πληρωθῇ ἡ ἡμέρα, καὶ αὕτη ἡ ὥρα ἐστίν.

Ἡ μέθοδος. ἐπειδὴ ϛ$^{ον}$ ιη$^{ον}$ προσέθηκε, ποίησον θ ιβ· γίνονται ρη· καὶ λῦσον εἰς ια· (διὰ τί δὲ εἰς ἕνδεκα; ἐπειδὴ ϛ$^{ον}$ ιη$^{ον}$ προσέθηκεν, ὅ ἐστιν ια θθ$^{α}$·) γίνονται οὖν τὸ ια$^{ον}$ τῶν ρη, θ 𐅵 δ$^{ον}$ κβ$^{ον}$ μδ$^{ον}$. εἰσὶν ὧραι θ 𐅵 δ$^{ον}$ κβ$^{ον}$ μδ$^{ον}$· <πρόσθες τὸ ϛ$^{ον}$ ιη$^{ον}$ τῶν θ 𐅵 δ$^{ον}$ κβ$^{ον}$ μδ$^{ον}$·> γίνονται β ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$· ὁμοῦ ιβ.

Ἡ ἀπόδειξις. Τὸ ϛ$^{ον}$ τῶν θ· γίνεται α 𐅵. καὶ τὸ ιη$^{ον}$ τῶν θ· 𐅵· ὁμοῦ γίνεται β. καὶ τὸ ϛ$^{ον}$ ιη$^{ον}$ τοῦ 𐅵 δ$^{ον}$ κβ$^{ον}$ μδ$^{ον}$· γίνεται ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$. καὶ τὸ ϛ$^{ον}$ ιη$^{ον}$ τοῦ 𐅵· γίνονται θ$^{ον}$. καὶ τὸ ϛ$^{ον}$ ιη$^{ον}$ τοῦ δ$^{ον}$· γίνεται ιη$^{ον}$. καὶ τὸ ϛ$^{ον}$ ιη$^{ον}$ τοῦ κβ$^{ον}$· γίνεται ϙθ$^{ον}$. καὶ τὸ ϛ$^{ον}$ ιη$^{ον}$ τοῦ μδ$^{ον}$· γίνεται ρϙη$^{ον}$· ὁμοῦ γίνονται θ$^{ον}$ ιη$^{ον}$ ϙθ$^{ον}$ ρϙη$^{ον}$· γίνεται ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$. τὸ θ$^{ον}$ τῶν ϙθ· γίνονται ια. καὶ τὸ ιη$^{ον}$ τῶν ϙθ· γίνονται ε 𐅵. τὸ

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[76] ι L
[77] πδ$^{ον}$ L
[78] Ἄλλο ἡ L

ϙθ$^{ον}$ τῶν ϙθ· γίνεται α. τὸ ρϙη$^{ον}$ τῶν ϙθ· γίνεται 𐅵· ὁμοῦ ιη. ιη εἰς ϙθ· ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$· ιβ, η δ$^{ον}$· κβ, δ 𐅵· λγ, γ· μδ, β δ$^{ον}$· συνήξαμεν ιη. εἰσὶν οὖν ὧραι θ 𐅵 δ$^{ον}$ κβ$^{ον}$ μδ$^{ον}$.

In another way the question.

Add 1/6 1/18 of the past hours in order that the day be completed, and this is the time it is.

Procedure. Since he turns out to add 1/6 1/18, do 9 ‹by› 12: they yield 108; and resolve into 11; (and why into eleven? Because he turns out to add 1/6 1/18, which ‹yielding› is 11/9:) then 1/11 of 108 yield 9 ½ ¼ 1/22 1/44. Then it is 9 ½ ¼ 1/22 1/44 o'clock; <add 1/6 1/18 of 9 ½ ¼ 1/22 1/44:> they yield 2 1/12 1/22 1/33 1/44: together 12.

Proof. 1/6 of 9: it yields 1 ½. And 1/18 of 9: ½: together it yields 2. And 1/6 1/18 of ½ ¼ 1/22 1/44: it yields 1/12 1/22 1/33 1/44. And 1/6 1/18 of ½: they yield 1/9. And 1/6 1/18 of ¼: it yields 1/18. And 1/6 1/18 of 1/22: it yields 1/99. And 1/6 1/18 of 1/44: it yields 1/198: together they yield 1/9 1/18 1/99 1/198: it yields 1/12 1/22 1/33 1/44. 1/9 of 99: they yield 11. And 1/18 of 99: they yield 5 ½. 1/99 of 99: it yields 1. 1/198 of 99: it yields ½: together 18. 18 into 99: 1/12 1/22 1/33 1/44. 12, 8 ¼; 22, 4 ½; 33, 3; 44, 2 ¼; we gathered 18. Then it is 9 ½ ¼ 1/22 1/44 o'clock.

**Problem 2**. The final check contains a further check, to the effect of proving that two sums of unit fractions are equal. Note the final list of parts of 99. A step was omitted by *saut du même au même*. *Equation*. $x + (a/b)x = k$, with $(a,b,k) = (2,9,12)$. *Algorithm*. $(a,b,k) \rightarrow bk \rightarrow bk/(b + a) = x$.

## 3

[*** = *Anonymus* 1306, item 3 of ψηφιφορικὰ προβλήματα πάνυ ὀφέλημα; cf. *Anonymus* J, no. c, e]

Ἄλλη ἐρώτησις.

Ἠρώτησε τίς τινὰ ποίᾳ ὥρᾳ ἐνεθυμήθη τί πρᾶξαι.

Ἡ μέθοδος. παρασκεύαζε τὸν ἐπερωτῶντα, ἥνπερ ὥραν ἐνεθυμήθη, διπλῶσαι αὐτὴν παρ' ἑαυτῷ, καὶ τὰ διπλωθέντα τριπλῶσαι, καὶ τὰ τριπλωθέντα πενταπλῶσαι, καὶ τὰ πενταπλωθέντα δεκαπλῶσαι, καὶ ἐρωτώμενος παρὰ σοῦ τὴν συναχθεῖσαν ὁμάδα εἰπεῖν καὶ τότε ταῦτα παρὰ σεαυτῷ λῦε εἰς τὰ τ, καὶ σκόπει ποίῳ ἀριθμῷ[79] ἀπηρτίσθη, καὶ εὑρήσεις τὴν ὥραν ἥνπερ ἐνεθυμήθη.

ὑποδείξεως χάριν, |[165v] Ἐνεθυμήθη τίς τρίτην ὥραν. ἀπαιτούμενος παρὰ σοῦ διπλῶσαι αὐτὴν ποιεῖ ϛ, εἶτα τριπλῶσαι ταῦτα ποιεῖ ιη, πάλιν ταῦτα πενταπλῶσαι ποιεῖ ϙ, ὁμοίως ταῦτα δεκαπλῶσαι συνῆξε λοιπὸν τὰ ὅλα ⁊· ταῦτα ἐκφαίνοντος ἐκείνου ἀνάλυε σὺ εἰς τ οὕτως· τριακόσιαι τρία· ⁊· ὡς δηλονότι τρίτῃ ὥρᾳ ἐνεθυμήθη τί ποιῆσαι. τούτῳ οὖν τῷ κανόνι ἀκολουθῶν πάσας ὥρας εὑρήσεις.

Another question.

Someone asked someone at what hour he intended to do something.

Procedure. Contrive the asker, that hour he indeed intended, to double it within himself, and to triplicate what has been doubled, and to quintuplicate what has been triplicated, and to decuplicate what has been quintuplicated, and asked by you to say the gathered collection, then also resolve these into 300 within yourself, and look at what number was completed, and you will find exactly the hour that he indeed intended.

For the sake of example, someone intended the third hour. Asked by you to double it he makes 6, afterwards to triplicate these he makes 18, again to quintuplicate these he makes 90, similarly to decuplicate these he finally gathered the whole 900; once that guy makes these manifest, you yourself, resolve out into 300 as follows: three hundreds ‹by› three: 900; so that clearly he intended to do something in the third hour. Then by following this rule you shall find all hours.

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[79] ῥυθμῷ L

**Problem 3**. A simple riddle in which the sought number is multiplied by a series of factors, whose product is cut off as a whole by the solver; asking the hour is just a pretext: no connection with probs. **1** and **2**. *Equation*. $a\times b\times c\times d\times x = k$ (the sign × denotes taking multiples), with $(a,b,c,d) = (10,5,3,2)$ and $k = 900$. *Algorithm*. $(a,b,c,d,k) \rightarrow k/abcd = x$.

**4**

[= *Anonymus* P, no. 64; cf. *Anonymus* V, no. 27]

Ἄλλη ἐρώτησις.

Λέγει τις κιστέρνα ἐστὶν ἔχουσα κρουνοὺς γ· ὁ εἷς κρουνὸς πληροῖ αὐτὴν διὰ μιᾶς ὥρας, ὁ β$^{ος}$ διὰ β, ὁ γ$^{ος}$ διὰ τριῶν ὡρῶν. τῶν τριῶν οὖν ὁμοῦ ἀφεθέντων διὰ πόσης ὥρας πληροῦσιν αὐτήν;

Ἡ μέθοδος. Ἐπειδὴ διὰ μιᾶς καὶ β καὶ γ εἶπεν ὡρῶν γεμίζειν τοὺς κρουνοὺς τὴν κιστέρναν, χρὴ εὑρεῖν τὸν ἀριθμὸν τὸν ἀπολύοντα 𐅵 γ$^{ον}$· ἔστιν οὖν ς· ποιοῦμεν οὖν ς α· ς· καὶ τὸ 𐅵 τῶν ς· γ. καὶ τὸ γ$^{ον}$ τῶν ς· β· ὁμοῦ γίνονται ια· ς εἰς ια· γίνεται 𐅵 κβ$^{ον}$· ὡς δηλονότι τῶν τριῶν ὁμοῦ ἐπιρεόντων γεμοῦσι τὴν κιστέρναν διὰ ὡρῶν 𐅵 κβ$^{ον}$. γνῶθι οὖν καὶ τοῦτο, ἕκαστος κρουνὸς πόσον μέρος πληροῖ τῆς κιστέρνης· ὁ γεμίζων διὰ μιᾶς ὥρας πληροῖ τῆς κιστέρνας μέρος 𐅵 κβ$^{ον}$, ὁ δὲ διὰ β ὡρῶν πληρῶν αὐτὴν γεμίζει μέρος δ$^{ον}$ μδ$^{ον}$, ὁ δὲ διὰ τριῶν γεμίζων αὐτὴν ἀπολυόμενος σὺν τοῖς ἄλλοις δυσὶ κρουνοῖς πληροῖ τῆς κιστέρνης μέρος ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$.

Another question.

Someone says there is a tank having 3 springs; the one spring fills it in one hour, the 2$^{nd}$ in 2, the 3$^{rd}$ in three hours. Then the three being allowed to release together, in how many hours do they fill it?

Procedure. Since he said the springs fill the tank full in one and 2 and 3 hours, one must find the number that resolves $^1/_2$ $^1/_3$ off: then it is 6; then we do 6 ‹by› 1: 6; and $^1/_2$ of 6: 3. And $^1/_3$ of 6: 2: together they yield 11; 6 into 11: it yields $^1/_2$ $^1/_{22}$; so that clearly, the three flowing together, they fill the tank full in $^1/_2$ $^1/_{22}$ hours. Then be also aware of this, each spring what part fills of the tank: the one filling it full in one hour fills the $^1/_2$ $^1/_{22}$ part of the tank, the one filling it in 2 hours fills the $^1/_4$ $^1/_{44}$ part full, the one filling it full in three, once resolved off with the other two springs, fills the $^1/_{12}$ $^1/_{22}$ $^1/_{33}$ $^1/_{44}$ part of the tank.

**Problem 4**. The classical problem of the tank filled by several sources; it amounts to a proportional partition of the unit; see the commentary on prob. **5**. The givens are the same as *AP* XIV.133, 135. *Equation*. $x/a + x/b + x/c = 1$, with $a{:}b{:}c = 1{:}2{:}3$. *Algorithm*. $(a,b,c) \rightarrow abc \rightarrow abc(1/a) = bc \mid (1/b)abc = ac \mid (1/c)abc = ab \rightarrow bc + ac + ab \rightarrow abc/(bc + ac + ab) = x$. The parts of the tank filled by the three sources are stated to be $x/a = bc/(bc + ac + ab)$, $x/b = ac/(bc + ac + ab)$, $x/c = ab/(bc + ac + ab)$, respectively.

**5**

[= *Anonymus* P, no. 71]

{marg. ψῆφος τῶν νομισμάτων}

Τίς τελευτῶν κατέλειπε τρεῖς υἱοὺς ἐάσας αὐτοῖς νομίσματα ρθ, καὶ τῷ μὲν πρώτῳ εἴασε γ$^{ον}$ μέρος, τῷ δὲ β$^{ῳ}$ δ$^{ον}$, τῷ δὲ γ$^{ῳ}$ ε$^{ον}$. εἰπεῖν τί ἑκάστῳ αὐτῶν ἁρμόττει ἐκ τῶν ρθ νομισμάτων.

Ἡ μέθοδος. Χρὴ εὑρεῖν τὸν ἀριθμὸν τὸν ἀπολύοντα τὰς φωνάς, ὅς ἐστιν ὁ ξ. τὸ οὖν γ$^{ον}$ δ$^{ον}$ ε$^{ον}$ τῶν ξ· γίνεται μζ, ἅπερ καὶ λύουσι τὴν ψῆφον. ποίησον οὖν τὸ γ$^{ον}$ τῶν ξ· γίνεται κ· τὰ κ ἐπὶ τὰ ρθ· γίνεται ͵βρπ· τούτων τὸ μζ$^{ον}$· γίνεται νομίσματα μς κεράτια δὲ θ ς$^{ον}$ μζ$^{ον}$ σπβ$^{ον}$. ὁμοίως τὸ δ$^{ον}$ τῶν ξ· γίνεται ιε· τὰ ιε ἐπὶ τὰ ρθ νομίσματα· γίνεται ͵αχλε· τούτων τὸ μζ$^{ον}$· γίνεται νομίσματα λδ κεράτια ιη 𐅵 δ$^{ον}$ η$^{ον}$ ϙδ$^{ον}$ ρπη$^{ον}$ <τος$^{ον}$>. τὸ [ς$^{ον}$ καὶ τὸ] ε$^{ον}$ τῶν ξ· γίνεται ιβ· τὰ ιβ ἐπὶ τὰ ρθ νομίσματα· γίνεται ͵ατη· τούτων τὸ μζ$^{ον}$· γίνεται νομίσματα κζ κεράτια ιθ 𐅵 γ$^{ον}$ ιε$^{ον}$ ϙδ$^{ον}$ σλε$^{ον}$. ὁμοῦ συμήχθησαν νομίσματα <ρθ>.

Ἄλλως ἡ μέθοδος. κράτει γ καὶ δ καὶ ε· γίνεται ιβ· ποίει[80] τὸ ιβ$^{ον}$ τῶν ρθ[81]· γίνεται θ ιβ$^{ον}$· τὰ θ ιβ$^{ον}$ ἐπανάβα[82] εἰς τὰ τρία· γίνεται κζ δ$^{ον}$. <καὶ τὰ θ ιβ$^{ον}$ ἐπανάβα εἰς δ· γίνεται λς γ$^{ον}$.> καὶ πάλιν τὰ θ ιβ$^{ον}$ ἐπανάβα εἰς ε· γίνεται με δ$^{ον}$ ς$^{ον}$[83]· ὁμοῦ συμήχθησαν νομίσματα ρθ.

Calculation of nomismata.

Someone dying left three sons bequeathing 109 nomismata to them, and he bequeathed a 3rd part to the first, a 4th to the 2nd, and a 5th to the 3rd. Say what is due to each of them of the 109 nomismata.

Procedure. One must find the number resolving the denominations off, which is 60. Then $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ of 60: it yields 47, which indeed also solve the calculation. Then do $\frac{1}{3}$ of 60: it yields 20; 20 by 109: it yields 2180; $\frac{1}{47}$ of these: it yields 46 nomismata and 9 $\frac{1}{6}$ $\frac{1}{47}$ $\frac{1}{282}$ carats. Similarly $\frac{1}{4}$ of 60: it yields 15; 15 by the 109 nomismata: it yields 1635; $\frac{1}{47}$ of these: it yields 34 nomismata 18 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{94}$ $\frac{1}{188}$ <$\frac{1}{376}$> carats. $\frac{1}{5}$ of 60: it yields 12; 12 by the 109 nomismata: it yields 1308; $\frac{1}{47}$ of these: it yields 27 nomismata 9 $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{15}$ $\frac{1}{94}$ $\frac{1}{235}$ carats: together <109> nomismata were gathered.

In another way the procedure. Keep 3 and 4 and 5: it yields 12; do $\frac{1}{12}$ of 1<0>9: it yields 9 $\frac{1}{12}$; mount 9 $\frac{1}{12}$ on three: it yields 27 $\frac{1}{4}$. <And mount 9 $\frac{1}{12}$ on 4: it yields 36 $\frac{1}{3}$.> And again mount 9 $\frac{1}{12}$ on 5: it yields 45 $\frac{1}{4}$ $\frac{1}{6}$: together 109 nomismata were gathered.

**Problems 5, 12, and 41**. Problems of proportional partition. Similar problems in Papyrus Achmin, nos. 3, 4, 10, 11, 13, 17, 47–49. In prob. **5** there are two solutions, according to whether the proportional parts are given as parts or as integers, respectively. Ambiguities of this kind can arise in the Greek numerical notation, as the system of signs discriminating cardinal and ordinal numerical letters (if any system is used) is unstable and prone to copying mistakes. It is likely that the double solution was conceived exactly as a reaction to this ambiguity. Add to this that the wording of the partition is a paradigmatic example of a formulaic clause whose meaning is different from its literal reading: the assigned parts are not the fractions of a whole (they do not add to 1), but the terms of the ratios between the assigned portions of the whole. A mere check-clause is provided at the end of both solutions. In probs. **12** and **41**, only the solution for integers is provided. Recall that 1 nomisma = 24 carats: thus, in the final calculation of the unknown number in each subroutine of the first solution, a rescaling must take place to carats of the residual fractional part of a nomisma; such residual fractions are $\frac{18}{47}$, $\frac{37}{47}$, and $\frac{39}{47}$, respectively. A step was omitted by *saut du même au même*. Note the verb form ἐπανάβα. **Solution 1**. *Equation*. $1/x + 1/y + 1/z = k$ and $x:y:z = a:b:c$, with $(a,b,c,k) = (\frac{1}{3},\frac{1}{4},\frac{1}{5},109)$. *Algorithm*. $(a,b,c,k) \rightarrow abc \rightarrow (1/a + 1/b + 1/c)abc$. $(1/a)abc \rightarrow [(1/a)abc]k \rightarrow [(1/a + 1/b + 1/c)abc][(1/a)abc]k = x$ | $(1/a)abc \rightarrow [(1/b)abc]k \rightarrow [(1/a + 1/b + 1/c)abc][(1/b)abc]k = y$ | $(1/c)abc \rightarrow [(1/c)abc]k \rightarrow [(1/a + 1/b + 1/c)abc][(1/c)abc]k = z$. **Solution 2**. *Equation*. $x + y + z = k$ and $x:y:z = a:b:c$, with $(a,b,c,k) = (3,4,5,109)$. *Algorithm*. $(a,b,c,k) \rightarrow a + b + c \rightarrow [1/(a + b + c)]k \rightarrow [1/(a + b + c)]ka = x$ | $[1/(a + b + c)]kb = y$ | $[1/(a + b + c)]kc = z$.

## 6

[***]

{marg. ἄλλως}

Ἰστέον ἐπὶ τῆς διανομῆς τῶν ρθ νομισμάτων ὅτι ὀφείλει τὸ γ$^{ον}$ ὑπερέχειν τοῦ μὲν δ$^{ου}$ ιβ$^{\varphi}$ τοῦ δὲ ε$^{ου}$ ιβ$^{\varphi}$ κ$^{\tilde{\varphi}}$. ὁμοίως καὶ τὸ δ$^{ον}$ ὀφείλει ὑπερέχειν τοῦ ε$^{ου}$ κ$^{\tilde{\varphi}}$. ὅσον οὖν μέρος ||[166r] γίνεται τὸ δ$^{ον}$ τοῦ γ$^{ου}$ τοσοῦτον μέρος καὶ τὰ γ$^{ον}$ τῶν δ$^{ων}$, καὶ πάλιν ὅσον μέρος γίνεται τὸ ε$^{ον}$ τοῦ δ$^{ου}$ τοσοῦτον γίνεται μέρος καὶ τὰ δ$^{ον}$ τῶν ε$^{ων}$.[84] χρὴ οὖν τὸ μεῖζον μέρος (ἤγουν τὸ γ$^{ον}$) λαμβάνειν πέντε νομίσματα τὸ δὲ μέσον <(ἤγουν τὸ δ$^{ον}$) δ τὸ δὲ ἔλαττον> (ἤγουν τὸ ε$^{ον}$) γ. καὶ ἵσταται ὁ ψῆφος ἀναμάρτητος.

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[80] ποιεῖ L
[81] ιθ L
[82] sic L
[83] καὶ L
[84] marg. οἶμαι τί σφάλλει

In another way.

One has to know that, in the distribution of the 109 nomismata, $^1/_3$ ought to exceed $^1/_4$ by $^1/_{12}$ and $^1/_5$ by $^1/_{12}$ $^1/_{20}$. Similarly $^1/_4$ ought also to exceed $^1/_5$ by $^1/_{20}$. Then, what part yields $^1/_4$ of $^1/_3$, such a part also yields $^1/_3$ of $^1/_4$, and again what part yields $^1/_5$ of $^1/_4$, such a part also yields $^1/_4$ of $^1/_5$. Then the greater part (namely, $^1/_3$) must take five nomismata, the middle <(namely, $^1/_4$) 4, and the lesser> (namely, $^1/_5$) 3. And the calculation stands faultless.

**Problem 6**. Remarks on the fractions involved in the previous problem, first solution. Nothing is wrong, contrary to what the marginal annotation "I think something has gone wrong" asserts. A step was omitted by *saut du même au même*.

**7**

[= *Anonymus* P, no. 72]

Λέγει τίς ἄλλῳ· λάβε τὸ ζ$^{ov}$ ὧν βαστάζω νομισμάτων καὶ δὸς τὸ δ$^{ov}$ ὧν βαστάζεις, καὶ ἔχομεν ἀπὸ λς νομίσματα. εἰπεῖν χρὴ ἀπὸ πόσων ἐβάσταζον νομισμάτων.

Ἐπειδὴ ζ καὶ δ εἶπον[85], κούφισον ἐκ τῶν ἀμφοτέρων ἀφ' ἑνός· λοιπὰ ς καὶ γ· τὸ ς$^{ov}$ τῶν λς· γίνεται ς· κούφισον ἐκ τῶν λς ς καὶ πρόσθες εἰς τὰ λς τὰ ς· γίνεται λ, μβ· τὸ γ$^{ov}$ τῶν μβ· γίνεται ιδ· κούφισον ἐκ τῶν μβ τὰ ιδ καὶ πρόσθες εἰς τὰ λ· ἰδοὺ μδ καὶ κη. εἶχεν οὖν ὁ εἷς νομίσματα μδ καὶ ὁ ἄλλος νομίσματα κη.

Ἡ ἀπόδειξις τῆς ψήφου· τὸ ζ$^{ov}$ τῶν κη· δ· δὸς τὰ δ τῷ ἔχοντι τὰ μδ, καὶ ἔχει ὁ εἷς μη καὶ ὁ ἄλλος κδ. δὸς τὸ δ$^{ov}$ τῶν μη (τὰ ιβ) τῷ ἔχοντι τὰ κδ, καὶ ἰδοὺ ἀμφότεροι ἔχουσιν ἀπὸ λς νομισμάτων.

Someone says to another one: take $^1/_7$ of the nomismata I hold and give $^1/_4$ of those you hold, and we have 36 nomismata each. One must say how many nomismata they held each.

Since they said 7 and 4, subtract one each from both of them: 6 and 3 as remainders; $^1/_6$ of 36: it yields 6; subtract 6 from 36 and add 6 to 36: it yields 30, 42; $^1/_3$ of 42: it yields 14; subtract 14 from 42 and add ‹them› to 30: there it is, 44 and 28. Then the one had 44 nomismata and the other 28 nomismata.

Proof of the calculation: $^1/_7$ of 28: 4; give 4 to the one having 44, and the one has 48 and the other 24. Give $^1/_4$ of 48 (namely, 12) to the one having 24, and there it is, both of them have 36 nomismata each.

**Problem 7**. A give-take problem with assigned exchange-fractions and equal, and assigned, final amount. One must intend that the second act of the give-take transaction takes place after the first is performed. A final check is provided. Note the distributive ἀπό. *Equation*. $x + y/a - (x + y/a)/b = k$, $y - y/a + (x + y/a)/b = k$, with $(a,b,k) = (7,4,36)$. *Algorithm*. $(a,b,k) \rightarrow (a-1,b-1) \rightarrow [1/(a-1)]k \rightarrow k \pm [1/(a-1)]k = \{[1/(a-1)](ak-2k), [1/(a-1)]k\} \rightarrow [1/(b-1)][1/(a-1)]k \rightarrow k \pm [1/(a-1)]k \mp [1/(b-1)][1/(a-1)]k = (y,x)$.

**8**

[***; cf. *Anonymus* L, no. a, b, d]

Ἄλλος τίς λέγει τινί· πάρεσχέ μοι δ νομίσματα ἐξ ὧν βαστάζεις, καὶ ἔχω διπλᾶ σου. λέγει ὁ ἄλλος· οὐχί, ἀλλὰ δός μοι δ νομίσματα αὐτὸς ἐξ ὧν βαστάζεις, καὶ ἔχομεν ἴσως. πόσα νομίσματα ἐβάσταζεν ὁ εἷς καὶ πόσα ὁ ἄλλος;

85 expect. εἶπε

Ἡ μέθοδος. Ἐπειδὴ διπλᾶ εἶπε, κράτησον ε καὶ ζ. καὶ ὅτι εἶπε δ νομίσματα δοῦναι ἀλλήλοις, ποίησον οὕτως. δ ε· κ· καὶ δ ζ· κη. ἐβάσταζεν οὖν ὁ εἷς νομίσματα κη καὶ ὁ ἄλλος κ.

Ἡ ἀπόδειξις. δὸς ἐκ τῶν κ νομισμάτων δ τῷ ἔχοντι τὰ κη, καὶ ἔχει ὁ εἷς νομίσματα λβ καὶ ὁ ἄλλος ις. ἰδοὺ διπλᾶ λάβει ἕκαστος τὰ ἑαυτοῦ κ καὶ κη. δώσει ὁ ἔχων τὰ κη νομίσματα δ τῷ ἔχοντι τὰ κ, καὶ ἔχουσιν οἱ δύο ἀπὸ κδ. ἰδοὺ ἴσα. ἐβάσταζεν, ὡς εἴπομεν, ὁ εἷς νομίσματα κ καὶ ὁ ἄλλος κη.

Someone says to someone: provide me with 4 nomismata from those you hold, and I have twofold as you. The other says: not at all, but give me 4 nomismata of those you yourself hold, and we have equally. How many nomismata did the one hold and how many did the other?

Procedure. Since he said twofold, keep 5 and 7. And as he said they gave 4 nomismata to one another, do as follows. 4 ‹by› 5: 20; and 4 ‹by› 7: 28. Then the one held 28 nomismata and the other 20.

Proof. From the 20 nomismata, give 4 to the one having 28, and the one has 32 nomismata and the other 16. There it is, each of them takes twofold the 20 and 28 of their own. The one having 28 nomismata will give 4 to the one having 20, and the two have 24 each: there it is, these are equal. The one, as we said, held 20 nomismata and the other 28.

**Problems 8, 10, 11**. Three give-take problems all solved in exactly the same way; prob. **11** does not work out a (impossible) solution because it applies the underlying insight when it could not be applied (a textual problem suggests that this drawback was perceived by some redactor or reviser). The exchange-amount and the final ratios are given; one of them is always the ratio of equality. Long final check. The statement "each of them takes twofold the 20 and 28 of their own" must not be taken at face value; it also occurs in the other give-take problems and must be a formulaic clause. Cf. *AP* XIV.145, 146. *Equation*. $(x + a)/(y - a) = k$ and $y + a = x - a$, with $(a,k) = (4,2), (4,3)$ in probs. **8** and **10**, respectively. Underlying insight: take the least numbers $(r,s)$ such that $r = s + 2$ and $(r + 1)/(s - 1) = k$, $k = 2, 3$ (probs. **8** and **10**, respectively); then rescale 1 to $a$ and $(r,s)$ accordingly: so that $(r,s,k) = (7,5,2), (5,3,3)$ in probs. **8** and **10**, respectively. The trick works with integer numbers only if $k = 2, 3$; it cannot work in the case of prob. **11** ($k = 4$), which in fact does not present any solution. *Algorithm*. $(a,k) \rightarrow (r,s)_k \rightarrow as = y \mid ar = y$.

**9**

[= *Anonymus* P, no. 73]

Διὰ ζ καὶ θ συστήσωμεν ὄνομα. Κόνων ἔχει ψήφους ϡϙ. ποίει οὕτως. ζ ϡ· ͵ςτ· θ ζ· ξγ· τὸ ξγ$^{ον}$ τῶν ͵ςτ· γίνεται ρ· τὰ ρ ἀνάλαβε εἰς θ· θ ρ· ϡ. οὕτω φέρει τὸ ὄνομα Κόνων διὰ ζ καὶ θ.

We shall build a name by means of 7 and 9. "Conon" has digits 990. Do as follows. 7 ‹by› 900: 6300; 9 ‹by› 7: 63; $^1/_{63}$ of 6300: it yields 100; take up 100 on 9: 9 ‹by› 100: 900. In this way the name "Conon" converts by means of 7 and 9.

**Problem 9**. A problem of onomatomancy. The Greek word Κόνων has digits 990 because 20(κ) + 70(ο) + 50(ν) + 800(ω) +50(ν) = 990. The rest of the text is pointless as it stands (it amounts to multiplying and dividing 900 by 63), and 90 appears nowhere. Maybe we should correct one of the two θ into a ϙ. For Greek onomatomancy, possibly in question here because of the reference to 7 and 9, see P. Tannery, Notice sur des fragments d'onomatomancie arithmétique. *Notices et extraits des manuscrits de la Bibliothèque Nationale* 31 (1886) 231–260, repr. Id., Mémoires scientifiques IX. Toulouse – Paris 1929, 17–50, and O. Neugebauer – G. Saliba, On Greek Numerology. *Centaurus* 31 (1989) 189–206.

### 10

[***]

Ἄλλη ἐρώτησις.

Λέγει τίς τῷ ἄλλῳ· δός μοι ἐξ ὧν ἔχεις νομίσματα δ, καὶ ἔχω σου τριπλᾶ. ὁ ἄλλος· οὐχί, ἀλλὰ δός μοι δ, καὶ ἔχομεν ἴσως. ἀπὸ πόσων ἐβάσταζον;

Ἡ μέθοδος. Ἐπειδὴ τριπλᾶ εἶπε, κράτει γ καὶ ε. καὶ ὅτι τέσσαρα προέτεινε, ποίησον δ γ· ιβ· καὶ δ ε· κ. ἐβάσταζεν οὖν νομίσματα ιβ καὶ ὁ ἄλλος κ.

δὸς ἐκ τῶν ιβ δ τῷ ἔχοντι τὰ κ, καὶ ἔχει ὁ εἷς νομίσματα κδ καὶ ὁ ἄλλος η. ἰδοὺ τριπλᾶ λάβουσι[86] τὰ ἴδια ιβ καὶ κ. δώσει ὁ ἔχων τὰ κ νομίσματα δ τῷ ἔχοντι τὰ ιβ, καὶ ἔχουσιν οἱ δύο ἀπὸ ις. ἰδοὺ ἴσα. εἶχεν οὖν, ὡς εἴπομεν, ὁ εἷς ιβ καὶ ὁ ἄλλος κ.

Another question.

Someone says to another one: give me 4 nomismata from those you have, and I have threefold as you. The other: not at all, but give me 4, and we have equally. How many ‹nomismata› did they hold each?

Procedure. Since he said threefold, keep 3 and 5. And as he offered four, do 4 ‹by› 3: 12; and 4 ‹by› 5: 20. Then he held 12 nomismata and the other one 20.

From 12, give 4 to the one having 20, and the one has 24 nomismata and the other 8. There it is, they take threefold their own 12 and 20. The one having 20 nomismata will give 4 to the one having 12, and the two have 16 each. There it is, these are equal. Then the one, as we said, held 12 and the other 20.

### 11

[***]

{marg. ἄλλη ἐρώτησις}

Λέγει ὁ εἷς τῷ ἄλλῳ· δός μοι ἐξ ὧν ἔχεις νομίσματα δ, καὶ ἔχω τετραπλᾶ σου. ὁ ἄλλος· οὐχί, |[166v] ἀλλὰ δός μοι δ, καὶ ἔχομεν ἴσως. πόσα ἕκαστος εἶχεν; ὅτι τετραπλᾶ εἶπεν, κράτει δ γ καὶ β γ, καὶ ποίησον κατὰ τὸν ἀνωτέρω κανόνα.

Another question.

The one says to the other: give me 4 nomismata from those you have, and I have fourfold as you. The other: not at all, but give me 4, and we have equally. How much did each of them have? As he said fourfold, keep 4 3 and 3 2, and do according to the above rule.

### 12

[***]

<Ψ>ῆφος τῶν ἐξαγίων.

Λέγει τίς ἐξάγια ἠγόρασα νομίσματος καὶ ἡμισίου [καὶ] καὶ τριμισίου. [φόλλης γ.] πόσου τὸ νόμισμα, πόσου τὸ ἥμισο, πόσου τὸ τριμίσιον ἀναλόγως;

Ἡ μέθοδος. Ποίησον κδ καὶ ιβ καὶ η· γίνονται μδ. τὰ μδ λύουσι τὴν ψῆφον. ἐπειδὴ ρκ λεπτῶν ἠγοράσθησεν τὰ ἐξάγια, ποίησον κδ ἐπὶ ρκ· γίνεται ͵βωπ· τούτων τὸ μδ$^{ον}$· γίνονται ξε γ$^{ον}$ ια$^{ον}$ λγ$^{ον}$. καὶ ιβ ἐπὶ ρκ· γίνεται ͵αυμ· τούτων τὸ μδ$^{ον}$· γίνονται λβ γ$^{ον}$ δ$^{ον}$ ια$^{ον}$ λγ$^{ον}$ μδ$^{ον}$. καὶ η ἐπὶ ρκ· γίνεται ϡξ· τούτων τὸ μδ$^{ον}$· γίνονται κα 𐅵 δ$^{ον}$ κβ$^{ον}$ μδ$^{ον}$· ὁμοῦ ρκ. ἔστιν οὖν τὸ νόμισμα νουμμίων ξε γ$^{ον}$ ια$^{ον}$

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[86] sic L

λγ$^{ον}$, καὶ τὸ ἥμισυ νουμμία λβ γ$^{ον}$ <δ$^{ον}$> ια$^{ον}$ λγ$^{ον}$ μδ$^{ον}$, καὶ τρίμισυν νουμμία κα 𐅵 δ$^{ον}$ κβ$^{ον}$ μδ$^{ον}$· ὁμοῦ συνήξαμεν νουμμία ρκ.

Calculation of exagia.

Someone says I bought exagia of a nomisma and semissis and tremissis. How much the nomisma, how much its half, how much the tremissis in proportion?

Procedure. Do 24 and 12 and 8: they yield 44. 44 solves the calculation. Since the exagia were bought at 120 parts, do 24 by 120: it yields 2880; $^1/_{44}$ of these: they yield 65 $^1/_3$ $^1/_{11}$ $^1/_{33}$. And 12 by 120: it yields 1440; $^1/_{44}$ of these: they yield 32 $^1/_3$ $^1/_4$ $^1/_{11}$ $^1/_{33}$ $^1/_{44}$. And 8 by 120: it yields 960; $^1/_{44}$ of these: they yield 21 $^1/_2$ $^1/_4$ $^1/_{22}$ $^1/_{44}$: together 120. Then the nomisma is of 65 $^1/_3$ $^1/_{11}$ $^1/_{33}$ noummia, and its half 32 $^1/_3$ <$^1/_4$> $^1/_{11}$ $^1/_{33}$ $^1/_{44}$ noummia, and the tremissis 21 $^1/_2$ $^1/_4$ $^1/_{22}$ $^1/_{44}$ noummia: together we gathered 120 noummia.

**Problem 12**. See the commentary on prob. **5**. A problem of proportional partition, with mere check-clause at the end. It is not easy to find a reason for the presence of φόλλης γ "of 3 folles" in the enunciation, as it does not figure in the subsequent computations. Maybe, together with the previous καί to be expunged, it is a misplaced and misread gloss ϛ φόλλεις γ, where we have to suppose a further misreading of a sign for φόλλις to a sign for μιλιαρέσιον. As a matter of fact, the follis was $^1/_{288}$ of a nomisma: Hendy, Coinage 26, and page 13 above. For the copper coin νουμμίον "noummion", here apparently taken to be $^1/_{120}$ of an exagion, see Hendy, Coinage 28; for the noummion in the *Palaia Logarikê*, see Svoronos, Recherches 80, and references therein. For the names of a half and a third of a nomisma, here affected by wild oscillations in spelling and the former largely disfigured, see the table edited on page 12. The problem is enunciated with fractional givens $(a,b,c) = (1,{}^1/_2,{}^1/_3)$, but the procedure is initialized by an input rescaled to (24,12,8). *Equation*. $x + y + z = k$ and $x{:}y{:}z = a{:}b{:}c$, with $(a,b,c,k) = (24,12,8,120)$. *Algorithm*. $(a,b,c,k) \rightarrow a + b + c \,.\, ak \rightarrow ak/(a + b + c) = x \mid bk \rightarrow bk/(a + b + c) = y \mid ck \rightarrow ck/(a + b + c) = z$.

## 13

[***]

{marg. ψῆφος τόκων}

Λέγει τίς νομίσματα ρ ἐδάνεισα ὑπὲρ τόκων μηνῶν ζ ἐπὶ 𐅵 ρ$^{ῆς}$. τί λάβω;

Ἡ μέθοδος. Τὸ ρ$^{ὸν}$ τῆς μονάδος· γίνεται ξ· τὸ 𐅵 τῶν ξ· γίνεται λ· τὰ λ τί ποιοῦσι τῆς μονάδος; <σ$^{ον}$.> ποίησον τὰ νομίσματα ἐπὶ τοὺς μῆνας, ὅ ἐστι ρ ἐπὶ ζ· γίνεται ψ· καὶ λῦσον εἰς τὰ σ· τὸ σ$^{ον}$ τῶν ψ· γίνεται γ 𐅵. ὤφειλε λαβεῖν ὁ δανείσας ὑπὲρ νομισμάτων ρ τόκον εἰς τοὺς ζ μῆνας ἐπὶ ρ$^{ῆς}$ τὸ 𐅵 νομίσματα γ 𐅵.

Calculation of interest.

Someone says I lent at interest 100 nomismata for 7 months at $^1/_2$ per cent. What do I take?

Procedure. $^1/_{100}$ of the unit: it yields 60; $^1/_2$ of 60: it yields 30; what do 30 make of the unit? <$^1/_{200}$.> Do the nomismata by the months, which is 100 by 7: it yields 700; and resolve into 200; $^1/_{200}$ of 700: it yields 3 $^1/_2$. The lender of 100 nomismata at $^1/_2$ per cent ought to take an interest of 3 $^1/_2$ nomismata for the 7 months.

**Problems 13–18**. Calculations of interest. Cf. Papyrus Achmin, nos. 26–28, 33–37, 44–46, where, however, the temporal dimension is absent. The basic relation is {*a*mount lent}{*m*onths}{interest *r*ate} = *i*nterest. Probs. **13–15** and **17**, **18** prescribe calculation of the interest, prob. **16** the amount lent, all other quantities being given. All amounts are in nomismata. Probs. **15** and **16** are complementary. For the basic monetary unit (here, the nomisma) being divided into 6000 parts, see page 12 above. With the exception of prob. **18**, the interest rate is preliminarily rescaled to a quantity such that the unit is 6000; the factor 100 in this number obviously derives from the standard per cent scale, the factor 60 accommodates for fractional interest rates. *Preliminary rescaling*. ${}^1/_{100}6000 = 60 \rightarrow r60 \rightarrow r60/6000 = {}^r/_{100}$. *Equation*. $amr = i$, the data and the unknown being in order from probs. **13** to **18**,

$(a,m,r,i) = (100,7,{}^{1}/_{2},x), (120,5,{}^{1}/_{3},x), (100,12,{}^{1}/_{4},x), (x,12,{}^{1}/_{4},3), (100,12,{}^{2}/_{3},x), (100,12,1,x)$. *Algorithm*. Probs. **13–15**, **17**, **18**: $(a,m,r) \rightarrow am \rightarrow ({}^{r}/_{100})am = x$. Prob. **16**: $(m,r,i) \rightarrow ({}^{100}/_{r})i \rightarrow ({}^{1}/_{m})({}^{100}/_{r})i = x$.

**14**

[***]

Ἄλλη ἐρώτησις.

Ἐδάνεισα νομίσματα ρκ ὑπὸ τόκων μηνῶν ε ἐπὶ ρ$^{\text{ῆς}}$ τὸ γ$^{\text{ον}}$. τί λάβω;

Ἡ μέθοδος. τὸ ρ$^{\text{ὸν}}$ τῆς μονάδος· γίνονται ξ· τὸ γ$^{\text{ον}}$ τῶν ξ· γίνεται κ· τὰ κ τί ποιοῦσι τῆς μονάδος; τ$^{\text{ον}}$. πολλαπλασίασον τὰ νομίσματα ἐπὶ τοὺς μῆνας, ὅ ἐστι ρκ ἐπὶ ε· γίνεται χ· καὶ λύεις εἰς τὸ τ· τὸ τ$^{\text{ον}}$ τῶν χ· γίνεται β. ὤφειλε λαβεῖν ὁ δανείσας ὑπὲρ νομισμάτων ρκ τόκον ὑπὲρ μηνῶν ε ρ$^{\text{ῆς}}$ τὸ γ$^{\text{ον}}$ νομίσματα β.

Another question.

I lent at interest 120 nomismata for 5 months at ${}^{1}/_{3}$ per cent. What do I take?

Procedure. ${}^{1}/_{100}$ of the unit: they yield 60; ${}^{1}/_{3}$ of 60: it yields 20; what do 20 make of the unit? ${}^{1}/_{300}$. Multiply the nomismata by the months, which is 120 by 5: it yields 600; and you resolve into 300; ${}^{1}/_{300}$ of 600: it yields 2. The lender of 120 nomismata at ${}^{1}/_{3}$ per cent ought to take an interest of 2 nomismata for 5 months.

**15**

[***]

{marg. Ἄλλη ἐρώτησις}

Ἐδάνεισα νομίσματα ρ ἐπὶ ρ$^{\text{ῆς}}$ τὸ δ$^{\text{ον}}$ ὑπὲρ τόκων μηνῶν ιβ. τί λάβω;

Ἡ μέθοδος. Τὸ ρ$^{\text{ὸν}}$ τῆς μονάδος· γίνονται ξ· τὸ δ$^{\text{ον}}$ τῶν ξ· γίνονται ιε· τὰ ιε τί ποιοῦσι τῆς μονάδος; υ$^{\text{ον}}$. πολλαπλασίασον τὰ νομίσματα ἐπὶ τοὺς μῆνας, ὅ ἐστι ρ ἐπὶ ιβ· γίνεται ͵ασ· λῦσον εἰς τὰ υ· τὸ υ$^{\text{ον}}$ τῶν ͵ασ· γίνονται γ. ὤφειλε δοθῆναι ὑπὲρ νομισμάτων ρ τόκον ὑπὲρ μηνῶν ιβ νομίσματα γ.

Another question.

I lent at interest 100 nomismata for 12 months at ${}^{1}/_{4}$ per cent. What do I take?

Procedure. ${}^{1}/_{100}$ of the unit: they yield 60; ${}^{1}/_{4}$ of 60: they yield 15; what do 15 make of the unit? ${}^{1}/_{400}$. Multiply the nomismata by the months, which is 100 by 12: it yields 1200; resolve into 400; ${}^{1}/_{400}$ of 1200: they yield 3. For 12 months, 3 nomismata for 100 nomismata ought to be given as interest.

**16**

[***]

{marg. Ἄλλη ἐρώτησις}

Λέγει τίς ἐδάνεισα καὶ ἔλαβον ὑπὲρ τόκων ὑπὲρ μηνῶν ιβ ἐπὶ ρ$^{\text{ῆς}}$ τὸ δ$^{\text{ον}}$ νομίσματα γ. ὑπὲρ πόσων οὖν νομισμάτων ἔλαβον τὰ γ νομίσματα;

Ἡ μέθοδος. Τὸ ρ$^{\text{ὸν}}$ τῆς μονάδος· γίνονται ξ· τὸ δ$^{\text{ον}}$ τῶν ξ· ιε· τὰ ιε τί ποιοῦσι τῆς μονάδος; υ$^{\text{ον}}$. τὰ υ ἐπὶ τὰ νομίσματα γ· γίνεται ͵ασ. καὶ ἐπειδὴ ὑπὲρ ιβ μηνῶν ἐδόθησαν τὰ γ νομίσματα, ποίησον ιβ$^{\text{ον}}$ τῶν ͵ασ· γίνεται ρ. ἐδόθησαν οὖν τὰ γ νομίσματα ἐπὶ ρ$^{\text{ῆς}}$ τὸ δ$^{\text{ον}}$ ὑπὲρ μηνῶν ιβ εἰς νομίσματα ρ.

Another question.

Someone says I lent at interest for 12 months at $^{1}/_{4}$ per cent and took 3 nomismata. Then for how many nomismata did I take the 3 nomismata?

Procedure. $^{1}/_{100}$ of the unit: they yield 60; $^{1}/_{4}$ of 60: 15; what do 15 make of the unit? $^{1}/_{400}$. 400 by the 3 nomismata: it yields 1200. And since the 3 nomismata were given for 12 months, do $^{1}/_{12}$ of 1200: it yields 100. Then the 3 nomismata for 12 months at $^{1}/_{4}$ per cent were given for 100 nomismata.

**17**

[***]

{marg. Ἄλλη ἐρώτησις}

νομίσματα ρ ἐπὶ ω ἑκατοστῆς ὑπὲρ μηνῶν ιβ. τί δίδοται;

τὸ ρ$^{ὸν}$ τῆς μονάδος· ξ· τὸ ω τῶν ξ· γίνεται μ· τὰ μ τῆς μονάδος |[167r] τί ποιοῦσι; ρν$^{ον}$. πολλαπλασίασον τὰ ρ ἐπὶ ιβ· γίνεται ͵ασ· καὶ λῦσον εἰς ρν· τὸ ρν$^{ον}$ τῶν ͵ασ· γίνεται η. δίδοται οὖν ὑπὲρ νομισμάτων ρ ἐπὶ τόκῳ ρ$^{ῆς}$ τὸ ω ὑπὲρ μηνῶν ιβ νομίσματα η.

Another question.

100 nomismata at $^{2}/_{3}$ per cent for 12 months. What is given?

$^{1}/_{100}$ of the unit: 60; $^{2}/_{3}$ of 60: it yields 40; what do 40 make of the unit? $^{1}/_{150}$. Multiply 100 by 12: it yields 1200; and resolve into 150; $^{1}/_{150}$ of 1200: it yields 8. Then for 100 nomismata at an interest rate of $^{2}/_{3}$ per cent for 12 months are given 8 nomismata.

**18**

[***]

νομισμάτων ρ ἐπὶ τελείας ἑκατοστῆς τί δίδοται ὑπὲρ μηνῶν ιβ;

ἐπειδὴ ρ$^{ὴν}$ τελείαν εἶπε, πολλαπλασίασον τὰ νομίσματα ἐπὶ τοὺς μῆνας· ρ ιβ· γίνεται ͵ασ· καὶ λῦσον εἰς τὰ ρ διὰ τὴν τελείαν ἑκατοστήν· τὸ ρ$^{ὸν}$ οὖν τῶν ͵ασ· γίνεται ιβ. ἔστιν οὖν ὁ τόκος τῶν ρ νομισμάτων ἐπὶ τελείας ρ$^{ῆς}$ ὑπὲρ μηνῶν ιβ νομίσματα ιβ.

What is given for 100 nomismata at a full per cent rate for 12 months?

Since he said full per cent rate, multiply the nomismata by the months: 100 ‹by› 12: it yields 1200; and resolve into 100 because of the full per cent rate; then $^{1}/_{100}$ of 1200: it yields 12. Then the interest of 100 nomismata at a full per cent rate for 12 months is 12 nomismata.

**19**

[= *Anonymus* P, no. 74]

{marg. Ἡ ψῆφος τοῦ ἀργυροῦ}

Ἔστι τὸ ἐξάγιον τοῦ ἀργοῦ χρυσοῦ – τουτέστι τῶν κδ κερατίων – κα. τῶν ς νομισμάτων τί λάβω;

Ἡ μέθοδος. Κράτει κδ· κούφισον κα· λοιπὰ γ· γ εἰς κα· γίνεται ζ$^{ον}$. ἔστιν οὖν ἑκάστῳ νομίσματι χάραγμα νόμισμα ἀργοῦ χρυσοῦ α ζ$^{ον}$. τὸ οὖν ζ$^{ον}$ ἐστι τῶν κδ κερατίων γ ς$^{ον}$ ζ$^{ον}$ ιδ$^{ον}$ κα$^{ον}$. ἐπειδὴ οὖν ς νομίσματα θέλεις, ποιοῦμεν ἅπαξ ς· […] τὸ ς$^{ον}$ ζ$^{ον}$ ιδ$^{ον}$ κα$^{ον}$ τῶν ς· γίνεται β ∸ ιδ$^{ον}$· ὡς γίνεται ὑπὲρ νομισμάτων […] η κεράτια ιγ ∸ [ζ$^{ον}$] ιδ$^{ον}$.

Calculation of white ‹gold›.

An exagion of white gold—that is, of 24 carats—is of 21 ‹carats› fine. What do I take of 6 nomismata?

Procedure. Keep 24; subtract 21: 3 as remainders; 3 into 21: it yields $\frac{1}{7}$. Then there is 1 $\frac{1}{7}$ of a white gold coined nomisma for each ‹gold› nomisma. Then $\frac{1}{7}$ of 24 carats is 3 $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{14}$ $\frac{1}{21}$. Then since you want 6 nomismata, we do once 6; […] $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{14}$ $\frac{1}{21}$ of 6: it yields 2 $\frac{1}{2}$ $\frac{1}{14}$; so that it yields […] 8 carats $\frac{1}{2}$ $\frac{1}{14}$ for 6 nomismata.

**Problems 19–24**. Problems on the value of alloy currencies with variable fineness. All of them apply the rule of three, probs. **19–20** indirectly, probs. **21–24** directly. A feature of these problems is that the carat is both a weight unit (for instance of white gold) and the unit of value expressing fineness, namely, the amount with respect to 24 of pure gold in an alloy. With the exception of prob. **23**, which is complementary to prob. **24**, here we are always given the fineness of an exagion (= 24 carats weight) of white gold, and we are asked to find the gold content of another amount, sometimes expressed in nomismata (19–20), sometimes in carats (21–24). Thus, the basic relation is {*fi*neness}:{24} = {gold *c*arats}:{*w*hite gold carats}. The syntagm χάραγμα νόμισμα denotes the intrinsic value of a nomisma as a coined piece and not in its nominal value as a unit of account; it is in fact a synonym of ὑπέρπυρον, the basic unit of the system. From Alexios I's (ruled 1081–1118) monetary reform on, the nomisma was of 20 $\frac{1}{2}$ carats fineness and worth 20 $\frac{3}{4}$ carats weight of pure gold (Hendy, Coinage 16–17), which is the value assumed in probs. **19** and **20**. For these problems, cf. Rhabdas' *Letter to Tzavoukhes*, in Tannery, Notice 148.1–150.14. Probs. **19**, **20**, **22**, **24**, **48** are directly formulated in the first person singular. The portion between asterisks in the algorithm below is badly represented in the problem. For since 6 nomismata do not allow exact division by 7, the text correctly resolves the nomisma into 24 carats, yielding 3 $\frac{3}{7}$ (as usual, the common fraction is expressed as a sum of unit fractions) after division by 7. Rescaling to 6 nomismata, the calculation goes awry but remains partly consistent; since any correction would restore the text arbitrarily, I refrained from doing this. A correct text should read as follows: "Then since you want 6 nomismata, we make once 6; <and 3 by 6: they yield 18; and> $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{14}$ $\frac{1}{21}$ of 6: it yields 2 $\frac{1}{2}$ $\frac{1}{14}$; so that it yields 6 nomismata 18 carats $\frac{1}{2}$ [$\frac{1}{7}$] $\frac{1}{14}$ for 6 nomismata". *Equation*. $f:24 = c:w$, the data and the unknown being in order from probs. **19** to **24**, $(f,24,c,w) = (21,24,6,x)$, $(21,24,7,x)$, $(18,24,x,19)$, $(18,24,30,x)$, $(x,24,16,30)$, $(\frac{4}{5},24,16,x)$. *Algorithm*. $(f,24,c) \rightarrow 24 - f \rightarrow (24 - f)/f$ *$\rightarrow 1c + [(24 - f)/f]c = x$*.

## 20

[= *Anonymus* P, no. 74]

{marg. Ἄλλη ἐρώτησις}

Ἔστι τὸ ἐξάγιον {signum et marg. κερατίων} <κα. τῶν> ζ νομισμάτων τί λάβω;

ἀπόθου κδ· λάβε κα· λοιπὸν γ· γ εἰς κα· γίνονται ζ$^{\text{ον}}$, ὅ ἐστιν ἑκάστου νομισμάτων νόμισμα α ζ$^{\text{ον}}$ ἀργυροῦ χρυσοῦ· ποιοῦμεν ἅπαξ ζ· καὶ τὸ ζ$^{\text{ον}}$ τῶν ἑπτά· ὁμοῦ η. γίνεται οὖν εἰς νομίσματα ζ <η> νομίσματα χρυσοῦ ἀργυροῦ.

Another question.

An exagion is of <21> carats fine. What do I take of 7 nomismata?

Keep away 24; take 21: 3 as a remainder; 3 into 21: they yield $\frac{1}{7}$, which is 1 $\frac{1}{7}$ of a white gold nomisma for each of the ‹gold› nomismata; we do once 7; and $\frac{1}{7}$ of 7: together 8. Then it yields <8> nomismata of white gold for 7 ‹gold› nomismata.

**Problem 20**. Note ἀπόθου with the meaning of κράτει. *Equation*. $f:24 = c:w$, with $(f,24,c,w) = (21,24,7,x)$. *Algorithm*. $(f,24,c) \rightarrow 24 - f \rightarrow (24 - f)/f \rightarrow 1c + [(24 - f)/f]c = x$.

## 21

[= *Anonymus* P, no. 75]

Ἄλλη ἐρώτησις

<Ἔ>στω τὸ ἐξάγιον {signum et marg. κερατίων} ιη. τὰ ιθ πόσου;

Ἡ μέθοδος. ιη ἐπὶ ιθ· γίνεται τμβ· λῦσον εἰς κδ· γίνεται ιδ δ$^{ον}$. ἔστιν οὖν ἡ τιμὴ τῶν ιθ κερατίων τοῦ ἀργοῦ χρυσίου κερατίων ιδ δ$^{ον}$.

Another question.

Let an exagion be of 18 carats fine. How much is 19?

Procedure. 18 by 19: it yields 342; resolve into 24: it yields 14 $\frac{1}{4}$. Then the value of 19 white gold carats is of 14 $\frac{1}{4}$ carats.

**Problem 21**. *Equation*. $f:24 = c:w$, with $(f,24,c,w) = (18,24,x,19)$. *Algorithm*. $(f,24,w) \rightarrow fw \rightarrow fw/24 = x$.

**22**

[= *Anonymus* P, no. 76]

{marg. ἄλλη ἐρώτησις}

<Τ>ὸ ἐξάγιον κερατίων ιη. εἰς τὰ λ κεράτια πόσον χρυσοῦ ἀργοῦ ἐπάρω;

Ἡ μέθοδος. κδ ἐπὶ λ· γίνονται ψκ· τούτων τὸ ιη$^{ον}$· γίνεται μ. ἔστιν οὖν τῶν λ κερατίων χρυσίου ἀργοῦ κεράτια μ.

Another question.

An exagion is of 18 carats fine. How much do I raise of white gold for 30 carats?

Procedure. 24 by 30: they yield 720; $\frac{1}{18}$ of these: it yields 40. Then ‹the amount› for 30 carats is 40 white gold carats.

**Problem 22**. *Equation*. $f:24 = c:w$, with $(f,24,c,w) = (18,24,30,x)$. *Algorithm*. $(f,24,c) \rightarrow 24c \rightarrow (1/f)24c = x$.

**23**

[= *Anonymus* P, no. 77]

<Ἀ>ργοῦ χρυσοῦ κεράτια λ εἰς κεράτια ις. τὸ ἐξάγιον πόσου;

Ἡ μέθοδος. Ἐπὶ κδ· γίνονται τπδ· τούτων τὸ λ$^{ον}$· γίνεται ιβ ∠ ε$^{ον}$ ι$^{ον}$. ἔστιν οὖν τὸ ἐξάγιον τῶν λ τοῦ χρυσίου κερατίων ιβ ∠ ε$^{ον}$ ι$^{ον}$.

30 carats of white gold for 16 carats. Of how much is an exagion fine?

Procedure. By 24: they yield 384; $\frac{1}{30}$ of these: it yields 12 $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{10}$. Then an exagion of 30 ‹white› gold carats is of 12 $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{10}$ fine.

**Problem 23**. The givens of probs. **23** and **24** are complementary. *Equation*. $f:24 = c:w$, with $(f,24,c,w) = (x,24,16,30)$. *Algorithm*. $(24,c,w) \rightarrow c24 \rightarrow (1/w)c24 = x$.

**24**

[= *Anonymus* P, no. 78]

Ἄλλη ἐρώτησις

Τὸ ἐξάγιον ιβ ∠ ε$^{ον}$ ι$^{ον}$. τῶν ις κερατίων τί λάβω;

Ἡ μέθοδος. ις ἐπὶ κδ· γίνονται τπδ· ταύτας ἀνάλυσον εἰς ιβ ∠ ε$^{ον}$ ι$^{ον}$· γίνεται λ. ἔστιν οὖν ἡ τιμὴ τῶν λ κερατίων κερατίων ις.

Another question.

An exagion is of 12 ½ ⅕ ⅒ ‹carats› fine. What do I take of 16 carats?

Procedure. 16 by 24: they yield 384; resolve these out into 12 ½ ⅕ ⅒: it yields 30. Then the value of 30 ‹white gold› carats is 16 carats.

**Problem 24**. *Equation*. $f{:}24 = c{:}w$, with $(f,24,c,w) = (4/5,24,16,x)$. *Algorithm*. $(f,24,c) \rightarrow c24 \rightarrow c24/f = x$.

**25**

[= *Anonymus* P, no. 79]

{marg. Ἡ ψῆφος τοῦ ἀργυροῦ}

<Ἡ> λίτρα τοῦ ἀργυροῦ νομίσματα ε ∸. ἡ οὐγγία πόσου;

Ἡ μέθοδος. δίπλωσον τὰ ε ∸, καὶ ποίησον ια. (διὰ τί δὲ διπλώσομεν; διὰ τὸ γίνεσθαι ε ∸ νομίσματα κεράτια ρλβ. τὸ οὖν ιβ$^{ον}$ τῶν ρλβ· γίνεται ια.) ἐὰν οὖν ἐστιν ἡ λίτρα νομίσματα ε ∸, τοῦ ἀργυροῦ ἔρχεται ἡ τιμὴ κεράτια ια[87]. τὸ δὲ γραμμὸν πόσου; ἐπειδὴ ἡ οὐγγία γράμματα ἔχει κδ, τὰ ια κεράτια ποίησον εἰς τὰ κδ· γίνεται γ$^{ον}$ η$^{ον}$.

Ἐὰν οὖν ἐστιν ἡ οὐγγία τοῦ ἀργυροῦ κεράτια ια, ἔρχεται ἡ τιμὴ τοῦ γραμμοῦ κεράτια γ$^{ον}$ η$^{ον}$. γίνεται γὰρ τὸ γ$^{ον}$ τῶν κδ, η, καὶ τὸ η$^{ον}$ τῶν κδ, γ· ὁμοῦ ια. τούτῳ οὖν τῷ κανόνι πάντα τὰ εἰς τὸν ἄργυρον εὑρήσεις, εἰς μὲν τὴν οὐγγίαν |[167v] διπλῶν τὴν τιμὴν τῆς λίτρας καὶ νοῶν αὐτὰ κεράτια, εἰς δὲ τὴν τιμὴν τοῦ γραμμοῦ λύοντα τὴν οὐγγίαν εἰς κεράτια εἰς τὰ κδ.

Calculation of silver.

A pound of silver 5 ½ nomismata. How much an ounce?

Procedure. Double 5 ½, and make 11. (And why did we double? Because of 5 ½ nomismata being 132 carats. Then 1/12 of 132: it yields 11.) Then if a pound be of 5 ½ nomismata, the value of silver amounts to 11 carats. And how much a gram? Since an ounce has 24 grams, do the 11 carats into 24: it yields ⅓ ⅛.

Then if an ounce of silver is 11 carats, the value of a gram amounts to ⅓ ⅛ carats. In fact, ⅓ of 24, 8, and ⅛ of 24, 3: together 11. Then by means of this rule you will find everything concerning silver, concerning an ounce by doubling the value of a pound and by conceiving them as carats, concerning the value of a gram by resolving an ounce into carats, namely, into 24.

**Problem 25**. Conversion of units of measurement: weights and currencies (contrary to probs. **19–24**, ἀργυρός denotes here a silver coin). A single application of the rule of three is required. The standard equivalences are 1 nomisma = 24 carats (currency) and 1 pound = 12 ounces, 1 ounce = 24 grams (weight). Thus, if an amount in pounds $p$ is worth $n$ nomismata, the same amount in ounces $o$ is worth $2n$ carats, and again, the same amount in grams $g$ is worth $2n/24$ carats. This much is stated in the general rule with which the problem ends. For these conversion problems, cf. Rhabdas' *Letter to Tzavoukhes*, in Tannery, Notice 150.18–154.2. Before the rule, a check is provided. Cf. prob. **27**. *Algorithm*. $(p,n) = (1,n) \rightarrow (o,2n) \rightarrow (g,2n/24)$.

**26**

[***]

{marg. ἄλλη ἐρώτησις}

Λέγει τίς καυκίνον χρυσέμπαστον λιτρῶν ι νομισμάτων ρ· ἡ λίτρα τοῦ ἀργυροῦ νομισμάτων ς· καὶ ἡ λίτρα τοῦ χρυσίου νομισμάτων οβ. εἰπεῖν τί ἔχει χρυσὸν τί ἀργυρόν.

---

87 ια e ιβ fecit. m.1

Ἡ μέθοδος. Ἐπειδὴ ς νομισμάτων εἶπε τὴν λίτραν τοῦ ἀργυροῦ εἶναι καὶ οβ τὴν λίτραν τοῦ χρυσίου, ποίησον οὕτως. τὸ ς$^{ον}$ τῶν οβ· γίνονται ιβ· ἀφ’ ὧν [ἐκ τῶν ιβ], α· λοιπὰ ια. καὶ ὅτι ι λίτρας ἔστησεν τὸ καυκίον, ποίησον ς ἐπὶ ι· γίνονται ξ· τὰ ξ κούφισον ἐκ τῶν ρ· λοιπὰ μ· τὰ ια$^{α}$ τῶν μ· γίνεται γ ∸ ια$^{ον}$ κβ$^{ον}$[88]· κούφισον ἐκ τῶν ξ καὶ θὲς εἰς τὰ μ. ἔχει οὖν χρυσοῦ νομίσματα μγ ∸ ια$^{ον}$ κβ$^{ον}$ καὶ ἀργοῦ νομίσματα νς γ$^{ον}$ λγ$^{ον}$ νομισμάτων ρ.

ἴδωμεν τί ποιεῖ ἡ λίτρα.

ὁ χρυσὸς ἔχει νομίσματα μγ κεράτια ιε δ$^{ον}$ μδ$^{ον}$, ὅ ἐστιν οὐγγίαι ζ γράμματα ς κεράτια γ δ$^{ον}$ μδ$^{ον}$, καὶ ὁ ἀργυρὸς νομίσμα<τα> νς κεράτια η[89] ∸ ς$^{ον}$ κβ$^{ον}$ ξς$^{ον}$, ὅ ἐστι λίτραι θ οὐγγίαι δ γράμματα ιζ κεράτια β γ$^{ον}$ <δ$^{ον}$> ια$^{ον}$ λγ$^{ον}$ μδ$^{ον}$.

ἴδωμεν εἰς τί συνάγει ὁ ἄργυρος ὑπὲρ λιτρῶν θ οὐγγιῶν δ γράμματα ιζ κεράτια β γ$^{ον}$ <δ$^{ον}$> ια$^{ον}$ λγ$^{ον}$ μδ$^{ον}$, νομίσματα νς κεράτια η ∸ ς$^{ον}$ κβ$^{ον}$ ξς$^{ον}$.

οὕτως. ἀργυροῦ λίτραι θ ἀπὸ νομισμάτων ς γίνονται νομίσματα νδ. καὶ ὑπὲρ οὐγγιῶν δ τῆς οὐγγίας κεράτια ιβ· γίνονται νομίσματα β. καὶ ὑπὲρ γραμμάτων ιζ τοῦ γραμμοῦ κεράτιον ∸· γίνονται κεράτια η ∸. καὶ ὑπὲρ κερατίων β τοῦ κερατίου τὸ ιβ$^{ον}$· γίνεται κεράτια τὸ ς$^{ον}$. καὶ ὑπὲρ τοῦ γ$^{ον}$ δ$^{ον}$ ια$^{ον}$ λγ$^{ον}$ μδ$^{ον}$· γίνεται κεράτια τὸ κβ$^{ον}$ ξς$^{ον}$· ὁμοῦ λίτραι θ ἀργυροῦ οὐγγίαι δ γράμματα […] ς κεράτια γ δ$^{ον}$ μδ$^{ον}$· γίνεται νομίσματα μγ κεράτια ιε δ$^{ον}$ μδ$^{ον}$· ὁμοῦ νομίσματα ρ συνήχθησαν ὑπὲρ τιμῆς ὅλου τοῦ καυκίου.

Another question.

Someone says a gold-pasted cup of 10 pounds for 100 nomismata; a pound of silver is of 6 nomismata; and a pound of gold of 72 nomismata. Say what does ‹the cup› have of gold and what of silver.

Procedure. Since he said a pound of silver is of 6 nomismata and a pound of gold of 72, do as follows. $\frac{1}{6}$ of 72: they yield 12; from 12, 1: 11 as remainders. And as the cup stood of 10 pounds, do 6 by 10: they yield 60; subtract 60 from 100: 40 as remainders; $\frac{1}{11}$ of 40: it yields 3 $\frac{1}{2}$ $\frac{1}{11}$ $\frac{1}{22}$; subtract from 60 and set to 40. Then of 100 nomismata it has 43 $\frac{1}{2}$ $\frac{1}{11}$ $\frac{1}{22}$ nomismata of gold and 56 $\frac{1}{3}$ $\frac{1}{33}$ of silver.

Let us see what does a pound make.

Gold has 43 nomismata 15 $\frac{1}{4}$ $\frac{1}{44}$ carats, which is 7 ounces 6 grams 3 $\frac{1}{4}$ $\frac{1}{44}$ carats, and silver 56 nomismata 8 $\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{22}$ $\frac{1}{66}$ carats, which is 9 pounds 4 ounces 17 grams 2 $\frac{1}{3}$ <$\frac{1}{4}$> $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{44}$ carats.

Let us see what does silver gather for 9 pounds 4 ounces 17 grams 2 $\frac{1}{3}$ <$\frac{1}{4}$> $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{44}$ carats, namely, 56 nomismata 18 $\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{22}$ $\frac{1}{66}$ carats.

As follows. 9 pounds of silver 6 nomismata each yield 54 nomismata. And for 4 ounces an ounce being worth 12 carats: they yield 2 nomismata. And for 17 grams a gram being worth $\frac{1}{2}$ carat: they yield 8 $\frac{1}{2}$ carats. And for 2 carats a carat being $\frac{1}{12}$: it yields $\frac{1}{6}$ carats. And for $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{11}$ $\frac{1}{33}$ $\frac{1}{44}$: it yields $\frac{1}{22}$ $\frac{1}{66}$ carats: together 9 pounds of silver 4 ounces […] 6 grams 3 $\frac{1}{4}$ $\frac{1}{44}$ carats: it yields 43 nomismata 15 $\frac{1}{4}$ $\frac{1}{44}$ carats: together 100 nomismata were gathered for the value of the whole cup.

**Problem 26**. Cf. probs. **5**, **12**, and **41**. Cf. *AP* XIV.11, 13. The problem sets out a cup of given weight made of gold and of silver. The nomismata gold and silver are worth are also given. One must find the amount of gold and of silver used in the cup, and their values in nomismata. The text sets the two values as unknown in the algorithm. The results, expressed in unit fractions as usual, are 43 $\frac{7}{11}$ and 56 $\frac{4}{11}$, respectively. To compute the weights, one must bear in mind the following relations. Silver: 1 pound = 6 nomismata, 1 ounce = 12 carats, 1 gram = $\frac{1}{2}$ carat, 1 carat (weight) = $\frac{1}{12}$ carat (nominal fineness). Gold, of course, is obtained by rescaling the previous ones by 12: 1 pound = 72 nomismata, 1 ounce = 6 nomismata = 144 carats, 1 gram = 6 carats, 1 carat (weight) = 1 carat (fineness). Calculating with these equivalences, one easily spots some copying mistakes and a lacuna that affects most of the long final check of the calculation of the weight of gold. *Equation*. $x + y = k$ and $x/a + y/b = h$, with $(a,b,k,h) = (6,72,100,10)$.

[88] ιβ$^{ον}$ L
[89] ιη L

*Algorithm*. (*a*,*b*,*k*,*h*) → (1/*a*)*b* → (1/*a*)*b* – 1 . *ah* → *k* – *ah* → {1/[(1/*a*)*b* – 1]}(*k* – *ah*) → *ah* – {1/[(1/*a*)*b* – 1]}(*k* – *ah*) = *x* . (*k* – *ah*) – {1/[(1/*a*)*b* – 1]}(*k* – *ah*) = *y* → *x*/*a* . *y*/*b*.

**27**

[***]

{marg. ψῆφος τοῦ ἀκρολίου}

Ἔστιν ἡ οὐγγία τοῦ ἀκρολίου νομισμάτων γ. τὸ γραμμὸν πόσου;

Ἡ μέθοδος. διπλῶς νόησον τὴν ψῆφον, ἵνα ἅπερ νομίσματά εἰσιν ἐν τῇ οὐγγίᾳ τοσαῦτα κεράτια ἐν τῷ γραμμῷ. ἔστιν οὖν ὑποδείξεως χάριν ἡ οὐγγία τοῦ ἀκρολίου νομισμάτων γ· καὶ τὸ γραμμὸν δηλονότι ἔρχεται γ.

Calculation of akrolion.

An ounce of first-fruits is of 3 nomismata worth. How much a gram?

Procedure. Conceive the calculation in two ways, in order that, how many nomismata there are in indeed in an ounce, so many carats there be in a gram. Then, for the sake of example, an ounce of first-fruits is of 3 nomismata; clearly a gram also amounts to 3.

**Problem 27**. A very simple conversion problem: since there are as many carats in a nomisma as grams in an ounce (namely, 24), the numbers expressing the values in nomismata or in carats of an ounce or of a gram of anything coincide, respectively. The term ἀκρόλιον or ἀκρόλειον is very poorly attested; I have chosen a meaning of ἀπαρχή, a synonym recorded by Byzantine lexicographers, that fits the context of the problem. Cf. prob. **25**.

**28**

[***]

{marg. ψῆφος τῶν μαργαριτῶν}

Ἰστέον ὅτι ἐστιν ὁ λεγόμενος στατὴρ τῶν μαργαριτῶν ψηφίων ξ. ἔστιν οὖν καὶ οὐγγία κερατίων ιβ. ὑποδείξεως χάριν κοκκία β στένοντα κεράτια κ· ἔστιν ὁ στατὴρ αὐτῶν οὐγγίαι ιβ – τουτέστι νομίσματα ς. πόσου τὰ β κοκκία;

<Ἡ> μέθοδος. Ποιοῦμεν κ κ· υ· τούτων τὸ ν$^{ον}$· γίνεται η· ὁμοῦ υη· ταύτας τὰς υη ἀνάλυε εἰς τὸν στατῆρα, ὅ ἐστι ξ· γίνεται ς ∸ ι$^{ον}$ ε$^{ον}$. ἔστιν οὖν ἡ τιμὴ αὐτῶν ὡς πρὸς οὐγγίας ιβ νομισμάτων μ ∸ ι$^{ον}$ ε$^{ον}$, ὡς γίνεσθαι τὴν τιμὴν ἀκριβῆ τῶν β κοκκίων νομισμάτων μ κερατίων ιθ ε$^{ον}$.

Calculation of pearls.

One has to know that the so-called stater of pearls is of 60 counting units. Then an ounce is also of 12 carats worth. For the sake of example, 2 grains balancing 20 carats; their stater is 12 ounces worth—that is, 6 nomismata. How much 2 grains?

Procedure. We do 20 ‹by› 20: 400; $\frac{1}{50}$ of these: it yields 8: together 408; resolve these 408 out into a stater, which is 60: it yields 6 $\frac{1}{2}$ $\frac{1}{10}$ $\frac{1}{5}$. Then their value with respect to 12 ounces is of 40 $\frac{1}{2}$ $\frac{1}{10}$ $\frac{1}{5}$ nomismata, so as to yield the exact value of 2 grains, 40 nomismata 19 $\frac{1}{5}$ carats.

**Problems 28–31**. Conversions of units of measurement; prob. **31** gives the rule. A bewildering set of problems; despite a general statement in prob. **31**, the rule applied can only be induced from the algorithm. The whole issue rests upon the participle στένον(τα), whose meaning is "to weigh" (*LBG*, *sub voce*), and which I translate "to balance". It is always question of *g*rains στένοντα *c*arats, the stater (which has 60 parts, taken as a parameter of the algorithm and apparently coinciding with ounces; for the stater, see Schilbach, Byzantinische Metrologie 282 *sub voce*) being given as *o* ounces, which are worth *o*/2 nomismata since 1 ounce is stated to be 12 carats (= $\frac{1}{2}$ nomisma) worth. It is required to find the nomisma-value of the assigned grains suitably transformed into parts of a stater;

this transformation, which involves squaring the grain-value and rescaling it by $\frac{51}{50}$, I have been unable to justify. In probs. **28** and **31**, I have translated ψηφίον as "counting unit" instead of "part". A final reduction from fractional parts of a nomisma to carats (1 nomisma = 24 carats) is performed. *Algorithm*. $(r,c,o) \rightarrow rr \rightarrow (\frac{1}{50})rr \rightarrow rr + (\frac{1}{50})rr \rightarrow [rr + (\frac{1}{50})rr]/60 \rightarrow o/2\{[rr + (\frac{1}{50})rr]/60\}$.

**29**

[***]

{marg. ἄλλη ἐρώτησις}

<Τρία κ>οκκία στένοντα κεράτια ιη τοῦ στατῆρος ὄντος τιμῆς οὐγγιῶν β, ὅ ἐστι νόμισμα α. τὰ τρία κοκκία πόσου;

Ἡ μέθοδος. Ποιοῦμεν ιη ἐπὶ ιη· γίνεται τκδ· τούτων τὸ ν$^{ον}$· γίνονται ς ε$^{ον}$ ϛ$^{ον}$ ι$^{ον}$ οε$^{ον}$· ὁμοῦ γίνονται τλ ε$^{ον}$ ϛ$^{ον}$ ι$^{ον}$ οε$^{ον}$· ταῦτα λῦσον εἰς τὰ ξ· γίνεται ε ∸ ρκε$^{ον}$. γίνεται ἡ τιμὴ νομισμάτων ε ∸ ρκε$^{ον}$, ὡς γίνεσθαι νομίσματα καθαρὰ ε κεράτια ιβ ϛ$^{ον}$ οε$^{ον}$ ρκε$^{ον}$ σν$^{ον}$. |[168r]

Another question.

Three grains balancing 18 carats the stater being of a value of 2 ounces, which is 1 nomisma worth. How much three grains?

Procedure. We do 18 by 18: it yields 324; $\frac{1}{50}$ of these: they yield 6 $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{10}$ $\frac{1}{75}$: together they yield 330 $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{10}$ $\frac{1}{75}$; resolve these into 60: it yields 5 $\frac{1}{2}$ $\frac{1}{125}$. It yields a value of 5 $\frac{1}{2}$ $\frac{1}{125}$ nomismata, so as to yield 5 pure nomismata 12 $\frac{1}{6}$ $\frac{1}{75}$ $\frac{1}{125}$ $\frac{1}{250}$ carats.

**30**

[***]

{marg. ἄλλη ἐρώτησις}

<Κ>οκκίνον α στένον κεράτια ι ἐστιν ὁ στατὴρ αὐτῶν οὐγγίαι κ, ὅ ἐστι νομίσματα ι.

ποιοῦμεν ι ι· ρ· ὧν τὸ ν$^{ον}$· γίνεται β· ὁμοῦ ρβ· ταῦτα εἰς τὰ ξ· γίνεται α ∸ ε$^{ον}$. ἔστιν οὖν ἡ τιμὴ ὡς πρὸς οὐγγίας κ νομισμάτων ιζ.

Another question.

1 grain balancing 10 carats their stater is 20 ounces worth, which is 10 nomismata.

We do 10 ‹by› 10: 100; of which $\frac{1}{50}$: it yields 2: together 102; these into 60: it yields 1 $\frac{1}{2}$ $\frac{1}{5}$. Then the value with respect to 20 ounces is of 17 nomismata.

**31**

[= *Anonymus* P, no. 80]

Ἄλλη ψῆφος συμβαλλομένη τοῖς ἀγοράζουσιν.

Συναναγαγὼν τὸν πολλαπλασιαμὸν τοῦ κερατισμοῦ μέριζε εἰς τὸ ποσὸν τῶν κοκκίων, καὶ τὸ ποσὸν εἰς ὃ ἀναλύονται ἐπίβαλε κατὰ τοῦ στατῆρος – τουτέστι τῶν ξ ψηφίων – καὶ εἴ τι ἀθροίσεις, ἐκεῖνο τὸ ποσὸν εἰς τὸν οὐγγιασμὸν φέρων εὑρήσεις εὐχερῶς τὸ τίμημα.

Another calculation occurring to merchants.

Gathering the multiplication of the carat-value, divide into the quantity of grains, and the quantity into which they are resolved out put upon according to the stater—that is, to the 60 counting units—

and if you will put something together, converting that quantity into ounce-value you shall easily find the valuation.

**32**

[= *Anonymus* P, no. 81]

Τὰ γ ιζιζ$^{α}$ καὶ θ ιθιθ$^{α}$ τί ποιοῦσι τῆς μονάδος;

Ἡ μέθοδος. Ἐπειδὴ γ εἶπε ιζιζ$^{α}$ καὶ θ ιθιθ$^{α}$, ποιοῦμεν οὕτως. τρεῖς εἰς ιζ$^{ον}$· γίνεται ιβ$^{ον}$ ιζ$^{ον}$ να$^{ον}$ ξη$^{ον}$. καὶ θ εἰς ιθ$^{ον}$· γίνεται δ$^{ον}$ ϛ$^{ον}$[90] λη$^{ον}$ νζ$^{ον}$ οϛ$^{ον}$· συνάγονται οὖν αἱ φωναὶ ∸ ιζ$^{ον}$ λη$^{ον}$ να$^{ον}$ νζ$^{ον}$ ξη$^{ον}$ οϛ$^{ον}$. τὰ γ οὖν ιζιζ$^{α}$ καὶ θ ιθιθ$^{α}$ ποιοῦσι τῆς μονάδος ∸ ιζ$^{ον}$ λη$^{ον}$ να$^{ον}$ νζ$^{ον}$ ξη$^{ον}$ οϛ$^{ον}$.

ιζ ιθ· τκγ· τούτων ιζ$^{ον}$· γίνεται ιθ. καὶ τὸ ιθ$^{ον}$· ιζ. τὰ γ ιθ[91]· γίνεται νζ. καὶ τὰ θ ιζ[92]· γίνεται ρνγ· ὁμοῦ σι. ταῦτα τὰ σι συγκρινόμενα πρὸς τὰ τκγ γίνεται ∸ ι$^{ον}$ κ$^{ον}$ ͵ϛυξ$^{ον}$. εἰκοσαπλούμενα γὰρ τὰ σι γίνεται ͵δσ, εἰκοσαπλούμενα δὲ τὰ τκγ γίνεται ͵ϛυξ. ταῦτα οὖν συγκρινόμενα τὰ ͵δσ πρὸς τὰ ͵ϛυξ συνιστῶσιν ∸ ι$^{ον}$ κ$^{ον}$ ͵ϛυξ$^{ον}$.

$^3/_{17}$ and $^9/_{19}$ what do they make of the unit?

Procedure. Since he said $^3/_{17}$ and $^9/_{19}$, we do as follows. Three into $^1/_{17}$: it yields $^1/_{12}$ $^1/_{17}$ $^1/_{51}$ $^1/_{68}$. And 9 into $^1/_{19}$: it yields $^1/_4$ $^1/_6$ $^1/_{38}$ $^1/_{57}$ $^1/_{76}$; then the denominations are gathered, namely, $^1/_2$ $^1/_{17}$ $^1/_{38}$ $^1/_{51}$ $^1/_{57}$ $^1/_{68}$ $^1/_{76}$. Then $^3/_{17}$ and $^9/_{19}$ make $^1/_2$ $^1/_{17}$ $^1/_{38}$ $^1/_{51}$ $^1/_{57}$ $^1/_{68}$ $^1/_{76}$ of the unit.

17 ‹by› 19: 323; $^1/_{17}$ of these: it yields 19. And $^1/_{19}$: 17. 3 ‹by› 19: it yields 57. And 9 ‹by› 17: it yields 153: together 210; these 210 compared to 323 yield $^1/_2$ $^1/_{10}$ $^1/_{20}$ $^1/_{6460}$. In fact, 210 twentuplicated yield 4200, and 323 twentuplicated yield 6460. Then these 4200 compared to 6460 conjure up $^1/_2$ $^1/_{10}$ $^1/_{20}$ $^1/_{6460}$.

**Problems 32–38**. Calculations with unit and common fractions. Cf. Papyrus Achmin, nos. 6–9, 12, 14–16, 18–25, 29–32, 38–40, 50. Probs. **32** and **33** compute $^3/_{17}$ + $^9/_{19}$ by means of three algorithms; probs. **34**–**36** transform, by means of identical algorithms, $^3/_7$ into thirteenths, $^3/_{13}$ into sevenths, and $^2/_3$ $^1/_7$ $^1/_{21}$ into elevenths, respectively; prob. **37** calculates $^2/_3 - {}^1/_{11} - {}^1/_{17}$. As for prob. **38**, see the commentary on it. *Algorithms of prob. **32***. 1) $(^a/_b, ^c/_d) \rightarrow a(^1/_b)$. $c(^1/_d) \rightarrow a(^1/_b) + c(^1/_d)$. This algorithm amounts to calculating an expansion in unit fractions of both fractions and then gathering the results; use is made of the fact that $^1/_2$ is $^1/_4$ $^1/_6$ $^1/_{12}$. 2) $(^a/_b, ^c/_d) \rightarrow bd \rightarrow (^1/_b)bd = d \mid (^1/_d)bd = b\ .\ ad\ .\ cd \rightarrow ad + cd \rightarrow (ad + cd)/bd = {}^a/_b + {}^c/_d$. Final check, expanding the fraction by 20.

**33**

[***]

{marg. Ἄλλως ἡ μέθοδος}

Ἐπειδὴ γ ιζιζ$^{α}$ καὶ θ ιθιθ$^{α}$ εἶπε, πολυπλασίασον τὰ ιζ ἐπὶ τὰ ιθ· γίνεται τκγ. ποιήσον τρία ιθ· γίνεται νζ. καὶ θ ἐπὶ ιζ· γίνεται ρνγ· ὁμοῦ σι· τὰ σι ποίησον εἰς τὰ τκγ· γίνεται ∸ ιζ$^{ον}$ λη$^{ον}$ να$^{ον}$ νζ$^{ον}$ ξη$^{ον}$ οϛ$^{ον}$. ∸ (ρξα ∸), ιζ$^{ον}$ (ιθ), λη$^{ον}$ (η ∸), να$^{ον}$ (ϛ γ$^{ον}$), νζ$^{ον}$ (ε ω), ξη$^{ον}$ (δ ∸ δ$^{ον}$), οϛ$^{ον}$ (δ δ$^{ον}$).

The procedure in another way.

Since he said $^3/_{17}$ and $^9/_{19}$, multiply 17 by 19; it yields 323. Do three ‹by› 19: it yields 57. And 9 by 17: it yields 153: together 210; do 210 into 323: it yields $^1/_2$ $^1/_{17}$ $^1/_{38}$ $^1/_{51}$ $^1/_{57}$ $^1/_{68}$ $^1/_{76}$. $^1/_2$ (161 $^1/_2$), $^1/_{17}$ (19), $^1/_{38}$ (8 $^1/_2$), $^1/_{51}$ (6 $^1/_3$), $^1/_{57}$ (5 $^2/_3$), $^1/_{68}$ (4 $^1/_2$ $^1/_4$), $^1/_{76}$ (4 $^1/_4$).

[90] καὶ L
[91] ιζιζ$^{α}$ L
[92] ιθιθ$^{α}$ L

**Problem 33**. Final check, by listing the indicated parts of *bd*. *Algorithm*. ($^a/_b$,$^c/_d$) → *bd* | *ad* | *cd* → *ad* + *cd* → (*ad* + *cd*)/*bd* = $^a/_b$ + $^c/_d$.

### 34

[= *Anonymus* P, no. 82]

ἰδοὺ καὶ διὰ βραχείας μεθόδου ἐπεδείξαμεν ἐπιλυούσας φωνάς· γ ζζ$^{α}$[93] πόσα ιγιγ$^{α}$ ποιοῦσιν;

Ποιοῦμεν οὕτως. γ ιγ· γίνονται λθ· καὶ λύομεν εἰς ζ· τὸ ζ$^{ον}$ τῶν λθ· γίνονται ε ∸ ιδ$^{ον}$. ἔστιν οὖν τὰ γ ζζ$^{α}$ ιγιγ$^{α}$ ε ∸ ιδ$^{ον}$.

There it is, we also showed ‹the› resolving denominations by means of a shorter procedure: how many thirteenths 3 sevenths do make?

We do as follows. 3 ‹by› 13: they yield 39; and we resolve into 7; $^1/_7$ of 39: they yield 5 $^1/_2$ $^1/_{14}$. Then $^3/_7$ are 5 $^1/_2$ $^1/_{14}$ thirteenths.

**Problem 34**. A copying mistake has occurred. *Algorithm*. ($^a/_b$,$^x/_d$) → *ad* → *ad*/*b* = *x*.

### 35

[***]

{marg. Ἄλλως}

Τὰ γ ιγιγ$^{α}$ πόσα ζζ$^{α}$; ποιοῦμεν γ ζ· κα· καὶ λύομεν εἰς ιγ· τὸ ιγ$^{ον}$ τῆς κα· γίνεται α ∸ ιγ$^{ον}$ κϛ$^{ον}$. ἔστιν οὖν τὰ γ ιγιγ$^{α}$ ζζ$^{α}$ α ∸ ιγ$^{ον}$ κϛ$^{ον}$.

In another way.

How many sevenths $^3/_{13}$? We make 3 ‹by› 7: 21; and we resolve into 13; $^1/_{13}$ of 21: it yields 1 $^1/_2$ $^1/_{13}$ $^1/_{26}$. Then $^3/_{13}$ are 1 $^1/_2$ $^1/_{13}$ $^1/_{26}$ sevenths.

**Problem 35**. *Algorithm*. ($^a/_b$,$^x/_d$) → *ad* → *ad*/*b* = *x*.

### 36

[= *Anonymus* P, no. 83]

Τὸ 𐅷 ζ$^{ον}$ κα$^{ον}$ πόσα ιαια$^{α}$ ποιοῦσιν;

Ἡ μέθοδος. <Ἐ>πειδὴ τὸ 𐅷 ζ$^{ον}$ κα$^{ον}$ ϛ εἰς ζ εἰσι, ποιοῦμεν ϛ ια· γίνεται ξϛ· καὶ λύομεν εἰς ζ· τὸ οὖν ζ$^{ον}$ τῶν ξϛ· γίνεται θ ϛ$^{ον}$ ζ$^{ον}$ ιδ$^{ον}$ κα$^{ον}$[94]. ιαια$^{α}$ θ ϛ$^{ον}$ ζ$^{ον}$ ιδ$^{ον}$ κα$^{ον}$, καὶ ὅσα τοιαῦτα οὕτω γίνεται.

How many elevenths do $^2/_3$ $^1/_7$ $^1/_{21}$ make?

Procedure. Since $^2/_3$ $^1/_7$ $^1/_{21}$ are 6 into 7, we do 6 ‹by› 11: it yields 66; and we resolve into 7; then $^1/_7$ of 66: it yields 9 $^1/_6$ $^1/_7$ $^1/_{14}$ $^1/_{21}$. 9 $^1/_6$ $^1/_7$ $^1/_{14}$ $^1/_{21}$ elevenths, and how many such are, thus it yields.

**Problem 36**. A copying mistake has occurred. *Algorithm*. ($^a/_b$,$^x/_d$) → *ad* → *ad*/*b* = *x*.

---

93 α γ ιζιζ$^{α}$ L

94 κδ$^{ον}$ L

**37**

[***]

ἐκ τοῦ διμοίρου ἐὰν ὑφέλῃς ια$^{ον}$ καὶ ιζ$^{ον}$, τί καταλείπεται;

ποίει οὕτως. ια ιζ· ρπζ· τὸ 𐅷 τῶν ρπζ· γίνεται ρκδ 𐅷. πάλιν ποίει ια καὶ ιζ· γίνεται κη· τὰ κη ὕφειλον ἐκ τῶν ρκδ 𐅷· μένουσι ϙς 𐅷· τὰ ϙς 𐅷 μέρισον εἰς ρπζ· γίνεται ε$^{ον}$ ϛ$^{ον}$ ια$^{ον}$ ρο$^{ον}$ ρπζ$^{ον}$. [τὸ δ$^{ον}$ ἐκ τοῦ 𐅷 κουφίσῃς[95] καὶ οὕτω ποίει.]

If from two-thirds you remove $^1/_{11}$ and $^1/_{17}$, what is left out?

Do as follows. 11 ‹by› 17: 187; $^2/_3$ of 187: it yields 124 $^2/_3$. Again, do 11 and 17: it yields 28; remove 28 from 124 $^2/_3$: they remain 96 $^2/_3$; divide 96 $^2/_3$ into 187: it yields $^1/_5$ $^1/_6$ $^1/_{11}$ $^1/_{170}$ $^1/_{187}$. [Subtract $^1/_4$ from $^2/_3$ and do as follows.]

**Problem 37**. The final clause is out of place, nor does it pertain to the subsequent problem. *Algorithm*. $(^a/_b, ^1/_d, ^1/_f) \rightarrow df \rightarrow (^a/_b)df \,.\, d + f \rightarrow (^a/_b)df - (d + f) \rightarrow [(^a/_b)df - (d + f)]/df$.

**38**

[***]

|[168v] μέθοδος δι' ἧς ὀφείλομεν συναθροῖσαι τὰ λεπτὰ τῆς μονάδος.

Ἰστέον ὅτι ἔχει τὸ ια$^{ον}$ τῆς μονάδος πισθομόρια, ἅπερ τινὲς μαλλία καλοῦσιν, γ$^{ον}$ ια$^{ον}$ λγ$^{ον}$, τὸ δὲ κβ$^{ον}$, γ$^{ον}$ δ$^{ον}$ ια$^{ον}$ λγ$^{ον}$ μδ$^{ον}$, τὸ δὲ μδ$^{ον}$ ἔχει γ$^{ον}$ λγ$^{ον}$, τὸ δὲ πη$^{ον}$, ιβ$^{ον}$[96] κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$. ὁμαδεύσωμεν τὰς φωνὰς τὰς εὐχερῶς ὑπὸ τῆς δεξίας κρατουμένας· οἷον ἔχομεν γ$^{ον}$ καὶ γ$^{ον}$ καὶ γ$^{ον}$ – τουτέστιν ἐκ τῆς λύσεως τοῦ ια$^{ον}$ καὶ κβ$^{ον}$ καὶ μδ$^{ον}$ – καὶ ἐκ τοῦ κβ$^{ου}$, δ$^{ον}$, καὶ ἐκ τοῦ πη$^{ου}$, ιβ$^{ον}$· ὁμοῦ συνήξαμεν α γ$^{ον}$. ἔλθωμεν καὶ ἐπὶ τὰς ἄλλας φωνάς. εἰσὶν οὖν αὗται ια$^{ον}$ λγ$^{ον}$ καὶ ια$^{ον}$ λγ$^{ον}$ μδ$^{ον}$ καὶ λγ$^{ον}$ καὶ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$. συναθροίσωμεν αὐτὰς οὕτως. κράτει τὸ ια$^{ον}$ α καὶ τὸ λγ$^{ον}$ γ$^{ον}$, καὶ <τὸ ια$^{ον}$ α καὶ τὸ λγ$^{ον}$ γ$^{ον}$> τὸ μδ$^{ον}$ δ$^{ον}$, καὶ πάλιν τὸ λγ$^{ον}$ γ$^{ον}$, καὶ κβ$^{ον}$ 𐅵 καὶ λγ$^{ον}$ γ$^{ον}$ καὶ τὸ μδ$^{ον}$ δ$^{ον}$. συνήχθησαν οὖν δ γ$^{ον}$· ταῦτα τὰ δ γ$^{ον}$ λῦσον εἰς ια· γίνεται γ$^{ον}$ (γ 𐅷) κβ$^{ον}$ (𐅵) ξϛ$^{ον}$ (ϛ$^{ον}$)· γίνεται γ$^{ον}$ κβ$^{ον}$ ξϛ$^{ον}$. μίξωμεν οὖν καὶ τὴν α γ$^{ον}$ τὴν συναχθεῖσαν ἐκ τῶν στερεῶν· ὁμοῦ συνάγονται ψῆφοι α 𐅷 κβ$^{ον}$ ξϛ$^{ον}$, ὡς δῆλον εἶναι ὅτι συνάγουσιν αἱ φωναί – τουτέστι τὸ γ$^{ον}$ ια$^{ον}$ λγ$^{ον}$ <, τὸ γ$^{ον}$ δ$^{ον}$ ια$^{ον}$ λγ$^{ον}$ μδ$^{ον}$, τὸ γ$^{ον}$ λγ$^{ον}$> καὶ τὸ ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$ – α 𐅷 κβ$^{ον}$ ξϛ$^{ον}$. τούτῳ οὖν τῷ κανόνι πάντα τὰ λεγόμενα πισθομόρια συναθροίζων εἴσῃ τὰς μεθόδους φιλοπόνως εὑρίσκειν φιλομαθέστατε.

Procedure by means of which we ought to put together the parts of the unit.

One has to know that $^1/_{11}$ has further parts than the unit, which indeed some call mallia, namely, $^1/_3$ $^1/_{11}$, $^1/_{33}$, and $^1/_{22}$, $^1/_3$ $^1/_4$ $^1/_{11}$ $^1/_{33}$ $^1/_{44}$, and $^1/_{44}$ has $^1/_3$ $^1/_{33}$, and $^1/_{88}$, $^1/_{12}$ $^1/_{22}$ $^1/_{33}$ $^1/_{44}$. Let us collect those denominations that easily kept on the right; for instance, we have $^1/_3$ and $^1/_3$ and $^1/_3$—that is, from the resolution of $^1/_{11}$ and $^1/_{22}$ and $^1/_{33}$—and from $^1/_{22}$, $^1/_4$, and from $^1/_{88}$, $^1/_{12}$: together we gathered 1 $^1/_3$. Let us also come to the other denominations. Then these are $^1/_{11}$ $^1/_{33}$ and $^1/_{11}$ $^1/_{33}$ $^1/_{44}$ and $^1/_{33}$ and $^1/_{22}$ $^1/_{33}$ $^1/_{44}$. Let us put them together as follows. Keep $^1/_{11}$ 1 and $^1/_{33}$ $^1/_3$ and <$^1/_{11}$ 1 and $^1/_{33}$ $^1/_3$> and $^1/_{44}$ $^1/_4$, and again $^1/_{33}$ $^1/_3$ and $^1/_{22}$ $^1/_2$ and $^1/_{33}$ $^1/_3$ and $^1/_{44}$ $^1/_4$: then 4 $^1/_3$ were gathered; resolve these 4 $^1/_3$ into 11: it yields $^1/_3$ (3 $^2/_3$) $^1/_{22}$ ($^1/_2$) $^1/_{66}$ ($^1/_6$): it yields $^1/_3$ $^1/_{22}$ $^1/_{66}$. Then let us also merge 1 $^1/_3$ gathered from the solid ‹numbers›: together 1 $^2/_3$ $^1/_{22}$ $^1/_{66}$ parts are gathered, so as to be clear that the denominations—that is, $^1/_3$ $^1/_{11}$ $^1/_{33}$ <, $^1/_3$ $^1/_4$ $^1/_{11}$ $^1/_{33}$ $^1/_{44}$, $^1/_{44}$ has $^1/_3$ $^1/_{33}$> and $^1/_{12}$ $^1/_{22}$ $^1/_{33}$ $^1/_{44}$—gather 1 $^2/_3$ $^1/_{22}$ $^1/_{66}$. Then assembling by means of this rule all the so-called further parts you will know industriously to find the procedures, you fondest of learning.

95 κουσῃς L

96 γ$^{ον}$ L

**Problem 38**. A most interesting problem, despite some copying mistakes. Apparently, the μαλλία (word unknown to the TLG) or τῆς μονάδος πισθομόρια "further parts than the unit" are the unit fractions in a given resolution of an assigned (unit) fraction into unit fractions, only the fractional part exceeding a unit being retained. It is obvious that the μαλλία here listed add to something greater than the assigned fraction, so that some rescaling must have occurred. In fact, the indicated sequences of unit fractions add to 16 times the corresponding assigned fractions; since $^{16}/_{11}$ is greater than 1, $^{5}/_{11}$ is retained. Thus, the μαλλία set out add to $^{5}/_{11}$, $^{8}/_{11}$, $^{4}/_{11}$, and $^{2}/_{11}$, in this order. All the μαλλία are systematically gathered, and the result is 1 $^{8}/_{11}$. I am unable to explain the presence of the denomination στερεός "solid ‹number›" in this context. The last sentence of the problem has a clear interlocutive value.

## 39

[= *Anonymus* P, no. 24 = Rhabdas, no. XIII] | alium atramentum

Λέγει τις ὅτι προέλαβε τινὰ στάδια ὅσα προέλαβε, καὶ ἄλλος εἰσελθὼν μετὰ ἡμέρας κ ἐν τῷ πλοίῳ αὐτοῦ ἐποίει καθ' ἡμέραν στάδια υ, καὶ ἔφθασεν αὐτὸν διὰ ἡμερῶν ξ. πόσα στάδια ἐποίει καθ' ἡμέραν ὁ ἐξελθὼν πρῶτος;

ποίει οὕτως. ξ υ· γίνεται β͵δ· ἐπανάλαβε[97] τὰ κ ἐπὶ τὰ ξ· γίνεται π· τὸ π^ον τῶν β͵δ· γίνεται τ· ὡς δηλονότι ἐποίει ὁ προεξελθὼν καθ' ἑκάστην ἡμέραν στάδια τ.

Someone says that he was ahead of someone how many stadia he was ahead by, and another one coming into in his route after 20 days made 400 stadia per day, and overtook him in 60 days. How many stadia per day made the one who set out first?

Do as follows. 60 ‹by› 400: it yields 2400; take up 20 in addition on 60: it yields 80; $^{1}/_{80}$ of 2400: it yields 300; so that clearly the one who set out before made 300 stadia per each day.

**Problems 39, 43, 44**. Standard pursuit problems. The gloss πρόσθες for the non-canonical ἐπανάλαβε suggests that L copied an annotated set of problems. In probs. **39** and **43**, the relation used is that speed by elapsed time yields run-distance; the run-distances are equated of two runners, the second moving later than the first. Thus, one gets $v_1t_1 = v_2t_2$, with $t_1 = t_2 + a$. In prob. **39**, one has to find $v_1$, in prob. **43**, $t_2$ . *Equation*. $v_1(t_2 + a) = v_2t_2$, with $(v_1,t_2,a,v_2) = (x,60,20,400)$. *Algorithm*. $(t_2,a,v_2) \rightarrow t_2v_2 \,.\, t_2 + a \rightarrow [1/(t_2 + a)]t_2v_2 = x$.

## 40

[= *Anonymus* P, no. 84 = Planudes, *Great Calculation*, 191.17–193.21 Allard]

Ἐν ἀρίστῳ μῆλα παρετέθησαν, καὶ ἐδόθη τῷ ἑνὶ μῆλον α καὶ τὰ ζζ^α τῶν μεινάντων μήλων, καὶ τῷ δευτέρῳ β καὶ τὰ ζζ^α τῶν μεινάντων μήλων, καὶ τῷ τρίτῳ γ καὶ τὰ ζζ^α τῶν μεινάντων μήλων, καὶ τῷ τετάρτῳ δ καὶ τὰ ζζ^α τῶν μεινάντων μήλων, καὶ τοῖς ὑπολοίποις τῶν ἀριστούντων ὁμοίως. εἰπεῖν χρὴ πόσοι οἱ ἀρισοῦντες ἦσαν καὶ πόσα τὰ μῆλα.

{marg. μέθοδος} ἐπειδὴ ζ^ον εἶπε, κρατοῦμεν ζ· ἐπαίρομεν ἕν· λοιπὰ ς· ἐξάπλωσον τὰ ς· γίνεται λς· ὡς δῆλον ὅτι ἦσαν οἱ ἀρισοῦντες ς καὶ τὰ μηλα λς.

Ἡ ἀπόδειξις. Ἐκ τῶν λς μήλων δὸς τῷ ἑνὶ ἕν· μένουσι λε· δὸς καὶ τούτων τὸ ζ^ον· γίνονται ὁμοῦ ς. ἰδοὺ ἔλαβεν ὁ εἷς μῆλα ς· λοιπὰ ἔμειναν μῆλα λ· ὁ β, δύο· λοιπὰ κη· τούτων τὸ ζ^ον· γίνεται δ· ὁμοῦ ς. καὶ ἔλαβεν ὁ δεύτερος ς· ἔμειναν μῆλα κδ· |[169r] ὁ γ^ος, γ· λοιπὰ ἔμειναν μῆλα κα· καὶ τούτων τὸ ζ^ον· γίνεται γ· ὁμοῦ ς. καὶ ἔλαβεν ὁ τρίτος ς· λοιπὰ ἔμειναν μῆλα ιη· ὁ τέταρτος, δ· λοιπὰ ιδ· καὶ τούτων τὸ ζ^ον· γίνεται β· ὁμοῦ ς. ἔλαβεν καὶ ὁ τέταρτος ς· λοιπὰ ἔμειναν μῆλα ιβ· ὁ ε^ος, ε· λοιπὰ ζ· καὶ τούτων τὸ ζ^ον· γίνεται α· ὁμοῦ ς. ἔλαβεν καὶ ὁ ε^ος ς· λοιπὰ ἔμειναν ς. ἔλαβεν καὶ ὁ ς^ος τὰ ς μείναντα μῆλα. ἦσαν οὖν οἱ ἀριστοῦντες ς καὶ τὰ μῆλα λς.

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[97] πρόσθες s.l. m.1

Apples were served up for breakfast, and 1 apple and the sevenths of the remaining apples were given to one, and 2 apples and the sevenths of the remaining apples to a second one, and 3 apples and the sevenths of the remaining apples to a third one, and 4 apples and the sevenths of the remaining apples to a fourth one, and similarly to the left over ones of those having the breakfast. One must say how many those having the breakfast were and how many the apples.

Procedure. Since he said $^1/_7$, we keep 7; we raise one: 6 as remainders; sextuplicate 6: it yields 36; so that it is clear that those having the breakfast were 6 and the apples 36.

Proof. From the 36 apples give one to the one: 35 remain; give also $^1/_7$ of these: together they yield 6. There it is, the one took 6 apples: 30 apples remained as remainders; the 2$^{nd}$ ‹took› two: 28 as remainders; $^1/_7$ of these: it yields 4: together 6. The second also took 6; 24 apples remained; the 3rd, 3: 21 remained as remainders; and $^1/_7$ of these: it yields 3: together 6. The third also took 6; 18 apples remained; the fourth, 4: 14 as remainders; and $^1/_7$ of these: it yields 2: together 6. The fourth also took 6; 12 apples remained as remainders; the 5$^{th}$, 5: 7 as remainders; and $^1/_7$ of these: it yields 1: together 6. The 5$^{th}$ also took 6; 6 as remainders. The 6$^{th}$ also took the remaining 6 apples. Then those having the breakfast were 6 and the apples 36.

**Problem 40**. A much-contrived yet classical riddle of iterative partition. Cf. prob. **45** and Papyrus Achmin, no. 13, 17. Contrary to prob. **45**, this problem is not conducive to generalization because this does not always allow for non-integer solutions. Just note in this connection that the only given number provided is 7: as a matter of fact, it is tacitly assumed that each participant gets the same share of apples; moreover, that there are 6 participants in the breakfast is forced by choosing 7 as the part to be given to each. A long check is provided. *Equation*. Iterative: $i + (x - k_{i-1} - i)/7 = k_i$, $\sum_i k_i = a$, $i = 1 \ldots n$, $k_0 = 0$, where $x$ is the number of apples and $n$ the number of participants. Find $x$ and $n$. *Algorithm*. $(^1/_7) \rightarrow 7 - 1 \rightarrow 6(7 - 1) = x$. It is simply stated that $n = 6$.

**41**

[= *Anonymus* P, no. 85]

πρός τινα εἰσῆλθον τρεῖς τινές, καὶ ἔπιον δροσάτον λίτραν α τραχίων τξ. ἔπιον δὲ οὕτως. ὁ εἷς γ, ὁ ἄλλος δ καὶ ὁ ἄλλος ε. εἰπεῖν τί ἑκάστῳ ἁρμόττει δοῦναι ἀναλόγως ὧν ἔπιον.

ποίησον οὕτως. γ καὶ δ καὶ ε· ὁμοῦ γίνονται ιβ. τρίπλωσον τὰ τξ· γίνεται ͵απ· τούτων τὸ ιβ$^{ον}$· γίνεται ϙ. καὶ ὅτι ἔπιεν δ, τετράπλωσον τὰ τξ· γίνεται ͵αυμ· ὧν τὸ ιβ$^{ον}$· γίνεται ρκ. ἔπιεν δὲ καὶ ὁ γ$^{ος}$ ε· πεντάπλωσον τὰ τξ· γίνεται ͵αω· ὧν τὸ ιβ$^{ον}$· γίνεται ρν· ὁμοῦ γίνεται ϙ καὶ ρκ καὶ ρν, ἅ εἰσι τξ.

Three guys came into at someone’s, and drank 1 pound of drink ‹for› 360 trachia. They drank as follows. The first 3, the other one 4, and the other one 5. Say what each of them is due to give in proportion to what they drank.

Do as follows. 3 and 4 and 5: together they yield 12. Triplicate 360: it yields 1080; $^1/_{12}$ of these: it yields 90. And as he drank 4, quadruplicate 360: it yields 1440; of which $^1/_{12}$: it yields 120. And the third also drank 5; quintuplicate 360: it yields 1800; of which $^1/_{12}$: it yields 150: together it yields 90 and 120 and 150, which are 360.

**Problem 41**. See the commentary on prob. **5**. For the small coin τραχίον, see Rhabdas in Tannery, Notice 148.8–9, stating that $^1/_{26}$ of a carat is worth $^2/_3$ of a trachion, which entails that 1 nomisma = 416 trachia. A problem of proportional partition, with final check. *Equation*. $x + y + z = k$ and $x{:}y{:}z = a{:}b{:}c$, with $(a,b,c,k) = (3,4,5,360)$. *Algorithm*. $(a,b,c) \rightarrow a + b + c$ . $ak \rightarrow [1/(a + b + c)]ak = x \mid bk \rightarrow [1/(a + b + c)]bk = y \mid ck \rightarrow [1/(a + b + c)]ck = z$.

**42**

[= *Anonymus* P, no. 86]

{marg. τὸ τῶν μελισσῶν}

Μέλισσαι εἰσελθοῦσαι ἐν τόπῳ ἔφαγον μέλιτος λίτρας ρ, καὶ κρατηθεῖσα μία καὶ θλιβεῖσα ἐξέβαλε ϛ$^{ov}$ ζ$^{ov}$ ιδ$^{ov}$ κα$^{ov}$ οὐγγίας. εἰπεῖν πόσαι μέλισσαι ἦσαν αἱ τὸ μέλι φαγοῦσαι.

{marg. μέθοδος} Ἐπειδὴ ϛ$^{ov}$ ζ$^{ov}$ ιδ$^{ov}$ κα$^{ov}$ οὐγγίας εἶπε φαγεῖν τὴν μέλισσαν, τὴν οὐγγίαν β γ$^{ov}$ μέλισσαι ἔφαγον. (διὰ τί δὲ δύο γ$^{ov}$· διὰ τὸ γίνεσθαι τὸ ϛ$^{ov}$ ζ$^{ov}$ ιδ$^{ov}$ κα$^{ov}$ τῶν ζ γ, τὸ δὲ γ$^{ov}$ τῶν ζ γίνεται β γ$^{ov}$.) ἐπεὶ οὖν ἡ λίτρα ἔχει οὐγγίας ιβ, ποίησον β γ$^{ov}$ ἐπὶ ιβ· γίνεται κη. ἔφαγον οὖν τὴν λίτραν μέλισσαι κη. καὶ ὅτι ρ λίτρας τοῦ μέλιτος ἔφαγον, ποίησον οὕτως. κη ἐπὶ ρ· γίνεται ͵βω· ὡς δηλονότι ἔφαγον τὰς ρ λίτρας μέλισσαι ͵βω.

The one of the bees.

Bees coming to a place ate 100 pounds of honey, and one of them caught and squeezed gave out $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{14}$ $\frac{1}{21}$ ounces. Say how many bees there were eating the honey.

Procedure. Since he said that a bee ate $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{14}$ $\frac{1}{21}$ ounces, 2 bees $\frac{1}{3}$ ate an ounce. (And why two $\frac{1}{3}$? Because of $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{14}$ $\frac{1}{21}$ of 7 yielding 3, and $\frac{1}{3}$ of 7 yields 2 $\frac{1}{3}$.) Then since a pound has 12 ounces, do 2 $\frac{1}{3}$ by 12: it yields 28. Then 28 bees ate a pound. And as they ate 100 pounds of honey, do as follows. 28 by 100: it yields 2800; so that clearly 2800 bees ate the 100 pounds.

**Problem 42**. An iterated application of the rule of three. If a bee eats $r/s$ ounces of honey, $s/r$ bees eat 1 ounce, $12(s/r)$ eat a pound (= 12 ounces), $[12(s/r)]n$ eat $n$ pounds. *Algorithm*. $(r/s,n) \rightarrow s/r \rightarrow (s/r)12 \rightarrow [(s/r)12]n$.

**43**

[= *Anonymus* P, no. 87]

Λέγει τίς δοῦλος ἔφυγε καὶ προέλαβε τὸν δεσπότην αὐτοῦ ἡμέρας δ· ἐποίει δὲ τὴν ἡμέραν ὁ δοῦλος μήλια κδ καὶ ὁ δεσπότης μήλια λ. διὰ πόσων ἡμερῶν ἔφθασεν αὐτὸν ὁ δεσπότης αὐτοῦ;

{marg. Ἡ μέθοδος} Ἐπειδὴ δ ἡμέρας προέλαβε ὁ δοῦλος <καὶ> ἐποίει μίλια κδ, ποίησον δ ἐπὶ κδ· γίνεται ϙϛ. καὶ ὅτι ὁ δεσπότης λ μίλια ἐποίει, κούφισον ἐκ τῶν λ τὰ κδ, ἅπερ ἐποίει ὁ δοῦλος· λοιπὰ ϛ· τὸ ϛ$^{ov}$ τῶν ϙϛ· γίνεται ιϛ. ἔφθασεν οὖν τὸν δοῦλον ὁ δεσπότης δι’ ἡμερῶν ιϛ.

Someone says a slave escaped and was 4 days ahead of his master; and the slave made 24 miles in a day and the master 30 miles. In how many days his master overtook him?

Procedure. Since the slave was 4 days ahead <and> made 24 miles, do 4 by 24: it yields 96. And as the master made 30 miles, subtract 24, which indeed the slave made, from 30: 6 as remainders; $\frac{1}{6}$ of 96: it yields 16. Then the master overtook the slave in 16 days.

**Problem 43**. See the commentary on prob. **39**. *Equation*. $v_1(t_2 + a) = v_2t_2$ with $(v_1,t_2,a,v_2) = (24,x,4,30)$. *Algorithm*. $(v_1,a,v_2) \rightarrow av_1 \,.\, v_2 - v_1 \rightarrow [1/(v_2 - v_1)]av_1 = x$.

**44**

[= *Anonymus* P, no. 88; cf. *Anonymus* V, no. 81, *Anonymus* U, no. 11]

Σκύλος ἀπελύθη ὀπίσω λαγοῦ, προέκοψε δὲ ὁ λαγὸς πηδήματα μ, καὶ οὕτως ἀπελύθη ὁ σκύλος ποιῶν ἐπάνω τοῦ λαγοῦ ιβ$^{ov}$ κβ$^{ov}$ λγ$^{ov}$ μδ$^{ov}$ μέρος τοῦ πηδήματος.[98] διὰ πόσων πηδημάτων ἔφθασεν ὁ σκύλλος τὸν λαγόν;

[98] marg. ext. ὅτι τὸ ιβ$^{ov}$

ἐπειδὴ μ πηδήματα προέλαβεν ὁ λαγὸς τὸν σκύλον ὁ δὲ σκύλλος ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$ μέρος προέτυπεν ἐπάνω τοῦ πηδήματος τοῦ λαγοῦ, ποίησον μ ἐπὶ ια· γίνεται υμ· (διὰ τί δὲ ἐπὶ ια; διὰ <τὸ> τὸ ἀριθμὸν εἶναι τὸ ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$ τῶν ια·) ποίησον τὸ ∠ τῶν υμ· γίνεται σκ· (διὰ τί δὲ τὸ ∠; ὅτι αἱ φωναὶ β εἰς ια εἰσὶν[99] |[169v] τὰ δὲ β ἀριθμὸς τῶν ἡμίσεων ἐστίν). ἔφθασεν οὖν ὁ σκύλλος τὸν λαγὸν διὰ πηδημάτων σκ.

οὕτως. τὸ ιβ$^{ον}$ τῶν σκ· γίνεται ιη γ$^{ον}$. καὶ τὸ κβ$^{ον}$ τῶν σκ· γίνεται ι. καὶ τὸ λγ$^{ον}$ τῶν σκ· γίνεται ς ω. καὶ τὸ μδ$^{ον}$ τῶν σκ· γίνεται ε· ὁμοῦ μ.

A hound was released after a hare, and the hare was in advance of 40 leaps, and the hound was so released as to make the $^1/_{12}$ $^1/_{22}$ $^1/_{33}$ $^1/_{44}$ part of a leap above and beyond the hare's. In how many leaps the hound overtook the hare?

Since the hare was 40 leaps ahead of the hound and the hound struk the $^1/_{12}$ $^1/_{22}$ $^1/_{33}$ $^1/_{44}$ part ‹of a leap› above and beyond a leap of the hare, do 40 by 11: it yields 440; (and why by 11? Because of the number being $^1/_{12}$ $^1/_{22}$ $^1/_{33}$ $^1/_{44}$ of 11;) do $^1/_2$ of 440: it yields 220; (and why $^1/_2$? Because the denominations are 2 into 11 and 2 is number of the halves). Then the hound overtook the hare in 220 leaps.

As follows. $^1/_{12}$ of 220: it yields 18 $^1/_3$. And $^1/_{22}$ of 220: it yields 10. And $^1/_{33}$ of 220: it yields 6 $^2/_3$. And $^1/_{44}$ of 220: it yields 5: together 40.

**Problem 44**. See the commentary on prob. **39**. This problem is framed in terms of sought *l*eaps and their parts, thus eliminating any reference to speed, time, and distance. The common fraction expressed in terms of unit fractions is $^2/_{11}$, which provides the canonical answer to the two questions. A final check is provided. Note the two *marginalia*, the first of which is misplaced; they identify the relevant unit sum of unit fractions. *Equation*. $l + a = l + (^r/_s)\, l$. *Algorithm*. $(r,s,a) \to as \to (^1/_r)as = l$.

**45**

[= *Anonymus* P, no. 89 = Rhabdas, no. XI]

τὸ τῶν προσαιτῶν.

Ἤτει τίς τινὰ προσαίτης, ὁ δὲ διδοὺς λέγει· ἐὰν διπλωθῶσιν ἅπερ βαστάζω, παρέχω σοι νουμμία λε, καὶ ἐγένετο οὕτως. ὁμοίως καὶ ἐπὶ δευτέρῳ οὕτως, καὶ παρέσχε καὶ αὐτῷ νουμμία λε. ὁμοίως καὶ ἐπὶ γ$^{ῳ}$, καὶ ἔλαβε καὶ αὐτὸς νουμμία λε, καὶ οὐδὲν ἔμεινε τῷ δεδωκότι τὴν εὐποιΐαν. τί οὖν πρότερον ἐβάσταζεν.

{marg. μέθοδος} Ἐπειδὴ διπλῶσαι εἶπε καὶ τρεῖς προσαῖται ἦσαν, ποίησον τὸ ∠ τῆς α· γίνεται ∠· καὶ τὸ ∠ τοῦ ∠· γίνεται δ$^{ον}$· καὶ τὸ ∠ τοῦ δ$^{ον}$· γίνεται η$^{ον}$· ὁμοῦ γίνεται ∠ δ$^{ον}$ η$^{ον}$· ποίησον ἄρτι τὸ ∠ δ$^{ον}$ η$^{ον}$ τῶν λε· γίνεται λ ∠ η$^{ον}$, οὕτως. τὸ ∠ τῶν λε· γίνεται ιζ ∠. τὸ δ$^{ον}$ τῶν λε· γίνεται η ∠ δ$^{ον}$. τὸ η$^{ον}$ τῶν λε· γίνεται δ δ$^{ον}$ η$^{ον}$· ὁμοῦ γίνεται λ ∠ η$^{ον}$. ταῦτα ἐβάσταζε τὸ πρότερον ὁ τὴν εὐποιΐαν διδούς.

Ἡ ἀπόδειξις. δίπλωσον τὰ λ ∠ η$^{ον}$· γίνεται ξα δ$^{ον}$· δὸς ἐξ αὐτῶν λε· λοιπὰ κς δ$^{ον}$· δίπλωσον ταύτας· γίνεται νβ ∠· δὸς λε· λοιπὰ ιζ ∠· δίπλωσον ταῦτα· γίνεται λε· δὸς καὶ τῷ τρίτῳ λε, καὶ οὐδὲν ὑπολείπεται. ὡς οὖν εἴπομεν, ἐβάσταζε τὸ πρῶτον νουμμία λ ∠ η$^{ον}$.

The one of the beggars.

Some beggar begs someone, and the one who gives says: if what I indeed hold were doubled, I provide you 35 noummia, and so happened. Similarly so also with a second ‹beggar›, and he also gave him 35 noummia. Similarly also with a third, and this one also took 35 noummia, and nothing remained to the one who had given the beneficence. Then what did he hold before?

[99] marg. inf. ὅτι τὸ ιβ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ μδ$^{ον}$ τῶν β ιαια$^{α}$ ἤγουν ς$^{ον}$ ξς$^{ον}$ τοῦ ὅλου

Procedure. Since he said “to double” and there were three beggars, do ½ of 1: it yields ½; and ½ of ½: it yields ¼; and ½ of ¼: it yields ⅛: together it yields ½ ¼ ⅛; do now ½ ¼ ⅛ of 35: it yields 30 ½ ⅛, as follows. ½ of 35: it yields 17 ½. ¼ of 35: it yields 8 ½ ¼. ⅛ of 35: it yields 4 ¼ ⅛: together it yields 30 ½ ⅛. These held before the one who gives the beneficence.

Proof. Double 30 ½ ⅛: it yields 61 ¼; give 35 out of them: 26 ¼ as remainders; double these: it yields 52 ½; give 35: 17 ½ as remainders; double these: it yields 35; also give 35 to the third one, and nothing is left over. Then, as we said, he held first 30 ½ ⅛ noummia.

**Problem 45**. A much-contrived yet classical riddle, as the title testifies. Cf. prob. **40**. A complete check is provided. For the noummion, see prob. **12**. *Equation*. $2^n(\ldots(2(2x - a) - a)\ldots) - a = 0$, yielding $x = (\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n})a$. *Algorithm*. $(a,n) \rightarrow \frac{1}{2}, \frac{1}{4} \ldots \frac{1}{2^n} \rightarrow \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} \rightarrow (\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n})a = x$.

**46**

[= *Anonymus* P, no. 90]

Τίς ἔσυρε ϛ$^{ον}$ ιγ$^{ον}$ κϛ$^{ον}$ λθ$^{ον}$ μέρος τοῦ τοξαρίου καὶ ἔκρουσε στρουθία η. ἐὰν ἔσυρε ὅλον, πόσα ἔμελλε κρούειν;

Ἡ μέθοδος. Ἐπειδὴ τὸ ϛ$^{ον}$ ιγ$^{ον}$ κϛ$^{ον}$ λθ$^{ον}$ δ ἐστιν <ιγ$^{α}$>, καὶ ὅτι η στρουθία εἶπε κροῦσαι αὐτόν, ποιοῦμεν η ιγ· γίνεται ρδ· καὶ λύομεν εἰς δ· τὸ οὖν δ$^{ον}$ τῶν ρδ· γίνεται κϛ. ἐφόνευσεν οὖν, εἰ ἔσυρε ὅλον τὸ τοξάριον, στρουθία κϛ.

τὸ γὰρ ϛ$^{ον}$ τῶν κϛ γίνεται δ γ$^{ον}$, καὶ τὸ ιγ$^{ον}$ τῶν κϛ γίνεται β, καὶ τὸ κϛ$^{ον}$ τῶν κϛ γίνεται α, καὶ τὸ λθ$^{ον}$ τῶν κϛ γίνεται ω· ὁμοῦ η.

Someone stretched the ⅙ $^1/_{13}$ $^1/_{26}$ $^1/_{39}$ part of a bow and pierced 8 birds. If he had stretched the whole of it, how many would he have pierced?

Procedure. Since ⅙ $^1/_{13}$ $^1/_{26}$ $^1/_{39}$ is $^4/_{\langle 13 \rangle}$, and as he said he had pierced 8 birds, we do 8 ‹by› 13: it yields 104; and we resolve into 4; then ¼ of 104: it yields 26. Then, if he had stretched the whole bow, he would have killed 26 birds.

In fact, ⅙ of 26 yields 4 ⅓, and $^1/_{13}$ of 26 yields 2, and $^1/_{26}$ of 26 yields 1, and $^1/_{39}$ of 26 yields ⅔: together 8.

**Problem 46**. Compare prob. **42**. A simple application of the rule of three. If a bow stretched for a part $r/s$ kills $n$ birds, the wholly stretched bow will kill $(r/s)n$. A final check is provided. *Algorithm*. $(r/s,n) \rightarrow ns \rightarrow ns/r$.

**47**

[***] | primum atramentum

<Λ>έγει τις σῖτος ἐπράθη τῷ νομίσματι μόδια κη[100]. τῶν θ μοδίων τί ἐδόθησαν;

ποιοῦμεν οὕτως. τὰ θ μόδια ἐπὶ τὰ κδ κεράτια τοῦ νομίσματος· γίνεται σιϛ· ταύτας λῦσον εἰς τὰ κη μόδια· γίνεται κεράτια ζ ∸ ζ$^{ον}$ ιδ$^{ον}$. οἱ θ μόδιοι κεράτια ζ ∸ ζ$^{ον}$ ιδ$^{ον}$.

Someone says grain was sold at 28 modii for a nomisma. What were they given for 9 modii?

We do as follows. The 9 modii by the 24 carats of a nomisma: it yields 216; resolve these into 28 modii: it yields carats 7 ½ ⅐ $^1/_{14}$. The 9 modii ‹are sold› at 7 ½ ⅐ $^1/_{14}$ carats.

**Problems 47, 48**. Simple applications of the rule of three entailing conversion of units of measurement, from nomisma to carats: a price is provided as *m*odii/*n*omisma and one is required to find what is given for some assigned amount of modii, or vice versa. The nomisma of the price must be resolved into 24 carats. *Equation*. $m{:}n = m_1{:}n_1$, the data and the unknown being $(m,n,m_1,n_1) = (28,24,9,x)$ and $(28,24,x,9)$, respectively. *Algorithm*. $(m,n,m_1) \rightarrow m_1n \rightarrow m_1n/m = x$.

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[100] κα L

**48**

[***]
{marg. ἄλλη ἐρώτησις}
τῷ νομίσματι μόδια κη. εἰς τὰ θ κεράτια πόσα λάβω;
τὰ θ κεράτια ἐπὶ τὰ κη μόδια· γίνεται σνβ· λῦσον εἰς τὰ κδ διὰ τὸ νόμισμα· τὸ οὖν κδ$^{ον}$ τῶν σνβ· γίνεται ι 𐅵. ὀφείλει λαβεῖν τῶν θ μοδίων ι 𐅵.

Another question.
28 modii for a nomisma. How much do I take for 9 carats?
The 9 carats by the 28 modii: it yields 252; resolve into 24 because of the nomisma; then ¹⁄₂₄ of 252: it yields 10 ½. One ought to take 10 ½ of the 9 modii.

**Problem 48**. See prob. **47**. *Algorithm*. $(m,n,n_1) \rightarrow n_1m \rightarrow n_1m/n = x$.

## EDITION, TRANSLATION, AND COMMENTARY OF *ANONYMUS* J

Vat. gr. 191, f. 261r

ἀρχὴ σὺν θεῷ διαφόρων ἐρωτημάτων
Beginning with God of various questions

**a**

[= *Anonymus* 1306, item 1 of μέθοδοι καθολικαί; cf. *Anonymus* L, no. 8, 10, 11]
<Ἐ>ρώτησε τίς πρὸς ἕτερον ὅτι δός μοι ἀφ’ ὧν βαστάζεις ἓν καὶ λάβε τέσσαρα ἐξ ἐμοῦ, καὶ ἐσμὲν ἶσα βαστάζοντες. ἀπεκρίθη ὁ ἄλλος· δὸς καὶ σὺ ἐμοὶ τέσσαρα καὶ λάβε ἕν, καὶ ἐσμὲν ἶσα.
μέθοδος. εἰπὲ δ δ· ις διὰ τὸ ζητῆσαι δ· τὸ ἥμισυ οὖν τῶν ις ἔστιν ὀκτώ· <πρόσθες οὖν εἰς μὲν τὰ η, γ, εἰς δὲ τὰ ἕτερα η κούφισον ἕτερα γ·> λοιπὸν οὖν ὁ μὲν εἶχε ε ὁ δὲ ἕτερος ια.
ἐὰν δὲ ἀπὸ τῶν ια ἐκβάλῃς δ καὶ προσθήσεις α, γίνονται ὀκτώ. ὁμοίως καὶ ἀπὸ τῶν ε ἐὰν ἐκβάλῃς α καὶ προσθήσεις δ, γίνονται ὀκτώ.

Someone asked another one: give me one from those you hold and take four from me, and we are holding the same. The other answered: you too, give me four and take one, and we are ‹holding› the same.
Procedure. Say 4 ‹by› 4: 16 because of searching 4; then a half of 16 is eight; <then add 3 to 8, and subtract other 3 to the other 8;> then finally the one had 5, the other 11.
And if you take 4 away from 11 and will add 1, they yield eight. And similarly if you take 1 away from 5 and will add 4, they yield eight.

**Problems a, b, d**. Give-take problems: assigned exchange amount and assigned final ratios (one of them always the ratio of equality; the other once equality and twice double). Prob. **a** is indeterminate because the two conditions coincide: any two numbers whose difference is 6 will work; the choice of 4 must be partly dictated by analogy with the general solution of such problems, in which the rescaling number is the exchange amount: cf. διὰ τὸ ζητῆσαι δ. An omitted sequence is supplied on the basis of *Anonymus* 1306. Cf. *AP* XIV.145, 146, and the commentary on prob. **8**. *Equation*. $x + a - b = y - a + b$, twice. *Algorithm*. $(a) \rightarrow aa \rightarrow aa/2 \rightarrow aa/2 + (a - b) = y$ . $aa/2 - (a - b) = x$.

**b**

[cf. *Anonymus* L, no. 8, 10, 11]

<Ε>ἶπε τίς πρὸς ἕτερον· δός μοι τόσα ἀφ’ ὧν βαστάζεις, καὶ ἐσμὲν ἶσα, ἢ λάβε ἐξ ἐμοῦ τὰ αὐτά, καὶ ἔχεις διπλά.

μέθοδος. <Κ>ράτει ἀεὶ δώδεκα, καὶ τὸν ἀριθμὸν ὃν ἐρωτᾷ σε πολλαπλασίαζε εἰς τὰ ιβ, εἶτα μέρισον τὸν πολλαπλασιασμὸν εἰς ιβ, καὶ ταῦτα πάλιν μέρισον εἰς τὰ δύο, καὶ ἐπίδος τῷ μὲν ἑνὶ δωδέκατα ἑπτὰ τῷ δὲ ἑτέρῳ ιβ$^{α}$ιβ$^{α}$[101] ε.

Someone said to another one: give me such-and-such from those you hold, and we are ‹holding› the same, or take the same from me, and you have the double.

Procedure. Always keep twelve, and multiply the number that he asked you by 12, afterwards divide the multiplication into 12, and again divide these into two ‹parts›, and give seven twelfths to the one and $^{5}/_{12}$ to the other.

**Problem b**. Cf. the commentary on prob. **8**. It provides the general rule for $k = 2$ (it uses τόσα for the unknown!): one must rescale $^{7}/_{12}$ and $^{5}/_{12}$ by twelve times the exchanged amount. A copying mistake occurs in the final clause. Prob. **d** gives an application of the rule. *Equation*. $(x + a)/(y – a) = 2$, $y + a = x – a$. *Algorithm*. $(a) \rightarrow a12 \rightarrow (^{7}/_{12})a12 = x$ . $(^{5}/_{12})a12 = y$.

**c**

[cf. *Anonymus* P, no. 100; *Anonymus* L, no. 3]

<Ἔ>λαχον[102] Πέτρος, Παῦλος καὶ Ἀνδρέας, καὶ ἐξέβαλεν ὁ μὲν Πέτρος τρία, ὁ Παῦλος πέντε καὶ ὁ Ἀνδρέας δύο· ὁμοῦ δέκα· δίπλασον ταῦτα· καὶ γίνονται κ· καὶ πρόσθες καὶ ε· καὶ γίνονται κε· τὰ ἀμφότερα πενταπλασιαζόμενα· γίνονται ρκε· καὶ δεκαπλασιαζόμενα· γίνονται ͵ασν· καὶ πάλιν δεκαπλασιαζόμενα· γίνονται α̈͵βφ. ἄρτι τρίπλασον τὰ τοῦ Πέτρου· καὶ γίνονται θ. καὶ ἐνναπλασίασον τὰ τοῦ Παύλου· καὶ γίνονται με. καὶ τὰ τοῦ Ἀνδρέου δεκαπλασίασον· καὶ γίνονται κ· ὁμοῦ τῶν τριῶν οδ. ὀφείλεις οὖν καθ’ ἑαυτὸν δεκαπλασιάζειν ἀεὶ τὰ ι, καὶ ὑφέλλειν αὐτὰ ἀπὸ τοῦ ἀριθμοῦ τῶν οδ – ἢ καὶ ἄλλου ἀριθμοῦ τοῦ γινομένου ἀπὸ τῆς ἑνώσεως τῶν τριῶν (ἤγουν τοῦ τριπλασιασμοῦ, τοῦ ἐνναπλασιασμοῦ καὶ τοῦ δεκαπλασιασμοῦ) – καὶ τὰ καταλιμπανόμενα κράτει, καὶ ὕφελλε ἀπ’ αὐτῶν ὅσας ἑπτάδας ἔχεις, καὶ νόει ὅτι ὁ πρῶτος τοσαῦτα ἐξέβαλεν. ὅσα δέ σοι περιττεύου<σι>ν ἀπὸ τοῦ ὑφειλμοῦ τοῦ ἑπτά, νόει ὅτι ἐξέβαλεν ὁ β$^{ος}$. ὅσα δέ σοι λείπει εἰς τὸν ἀριθμὸν τοῦ ὅλου λαχίου τῶν τριῶν ἐξέβαλεν ὁ τρίτος.

Peter, Paul, and Andrew cast lots, and Peter threw three, Paul five, and Andrew two; together ten; double these; and they yield 20; and also add 5; and they yield 25; both of them quintuplicated; they yield 125; and decuplicated; they yield 1250; and again decuplicated; they yield 12500. Now triplicate those of Peter; and they yield 9. And ennuplicate those of Paul; and they yield 45. And decuplicate those of Andrew; and they yield 20; the three together 74. Then you always ought to decuplicate 10 by yourself, and remove <from> them [from] the number of 74—or even [from] another number yielded by the union of the three (namely, of the triplication, the ennuplication, and the decuplication)—and keep what is left out, and remove as many heptads as you have from them, and conceive that the first threw this much. And as much as remains over for you from the removal of seven, conceive that ‹this much› threw the 2nd. And as much as is left for you as far as the number of the whole casting of the three, ‹this much› threw the third.

[101] ις$^{α}$ις$^{α}$ J
[102] ἔλαχαν J

**Problem c**. Casting lots by dice: three people, two different prescriptions; what is given is the sum of the three castings and, in the second prescription, a suitable (and fixed) linear combination of them. In the second prescription, one of the 10s referred to is a parameter (cf. “always”), the other is the sum of the three castings, as derived from the previous relation. The subsequent step mistakenly interchanges subtrahend and minuend. Problems **c** and **e** are of the same kind. *Equations*. $10\{10[5(2\{x + y + z\} + 5)]\} = 10000\{x + y + z\} + 2500$ and $10(x + y + z) - (3x + 9y + 10z) = 7x + y$, which of course are identities. *Algorithms*. No algoritm is provided for the first prescription. The second: $(x + y + z, 3x + 9y + 10z) = (k,h) \to 10k \to 10k - h \to [(10k - h)/7] = x \to 10k - h - 7x = y \to k - x - y = z$. Here, $[x]$ is the integral part of $x$.

## d

[cf. *Anonymus* L, no. 8, 10, 11]

<Δ>ύο τινὲς ἠρώτησεν εἷς πρὸς τὸν ἕτερον· δός μοι ἀφ’ ὧν βαστάζεις ζ, καὶ ἔχω διπλάσιον[103], ἢ ἆρον ἐξ ἐμοῦ ζ, καὶ ἐσμὲν ἶσα βαστάζοντες.

μέθοδος. <Ε>ἴ τι ἂν ἐστὶν ὁ ἀριθμός, δωδεκαπλασίασον, ἤγουν ζ ιβ· πδ. εἶθ’ οὕτως πάλιν πενταπλασίαζε τὸν ἐκφωνούμενον ἀριθμόν (ἤγουν τὰ ζ), καὶ λέγε ε ζ· λε. λοιπὸν οὖν λε εἶχεν ὁ εἷς· ἐξερχομένων δὲ τῶν λε ἀπὸ τῶν πδ καταλιμπάνονται μθ. καὶ εἶχε ταῦτα τὰ μθ ὁ ἕτερος· ζ ζ γὰρ ἐστὶ μθ.

Two guys; the one asked to the other: give me 7 from those you hold, and I have the double, or raise 7 from me, and we are holding the same.

Procedure. If the number is something, dodecuplicate ‹it›, namely, 7 ‹by› 12: 84. Afterwards again, quintuplicate as follows the uttered number (namely, 7), and say 5 ‹by› 7: 35. Then finally the one had 35; and 35 coming out of 84, 49 are left out. And the other had these 49, for 7 ‹by› 7 is 49.

**Problem d**. General rule in prob. **b**. *Equation*. $(x + a)/(y - a) = 2$, $y + a = x - a$, with $a = 7$. *Algorithm*. $(a) \to a12 \,.\, a7 = x \to a12 - a7 = y$.

## e

[cf. *Anonymus* V, no. 38 = Spingou, Πῶς δεῖ εὑρίσκειν; *Anonymus* L, no. 3]

<Τ>οῦ δακτυλιδίου τῶν παίδων

<Κ>ράτει ἀριθμὸν οἷον θέλεις καὶ δίπλασον· πρόσθες ἐπ’ αὐτῷ ε· καὶ αὖθις πενταπλασίασον τὰ ὅλα· καὶ αὖθις τὰ ὅλα δεκαπλασίασον· πρόσθες ἐπ’ αὐτοῖς τὸν ἀριθμὸν τῶν δακτύλων· καὶ αὖθις τὰ ὅλα δεκαπλασίασον· καὶ ἴδε τὰ ὅλα· καὶ ὕφελλε ἀπ’ αὐτῶν πάντοτε ͵βφ· καὶ κράτησον τὰ ἀπομείνοντα. καὶ ὅσας μὲν χιλιάδας ἔχεις, ἔστιν ὁ ἀριθμὸς τοῦ λαοῦ· ὅσας δὲ δεκάδας, ἔστιν ὁ ἀριθμὸς τῶν δακτύλων. διὰ τί δὲ ὑφέλλεις ͵βφ; διότι ἡ ἀρχὴ ἔστι τὸ α· λοιπὸν οὖν διπλάζοντες τὸ α γίνεται β· προστιθέντες δὲ ε γίνεται ζ· πενταπλασιαζόμενα γοῦν γίνεται λε· δεκαπλασιαζόμενα γίνεται τν· καὶ πάλιν δεκαπλασιαζόμενα γίνεται ͵γφ. λοιπὸν οὖν ἓν ὀφείλοντες γυρεύειν λέγομεν· ἆρον τὰ ͵βφ· καὶ καταλιμπάνονται ͵α, ἥτις χιλιὰς ἐστὶ τοῦ ἑνός.

Of the ring of the boys.

Keep such a number as you like and double ‹it›; add 5 to it; and quintuplicate anew the whole; and decuplicate anew the whole; add the number of the fingers to them; and decuplicate anew the whole; and see the whole; and always remove 2500 from it; and keep what remains. And as many thousands you have, they are the number of people, and as many decads, they are the number of the fingers. Why do you remove 2500? Because the beginning is 1; then finally doubling 1 it yields 2; and adding 5 it yields 7; then quintuplicated they yield 35; decuplicated they yield 350; and again decuplicated

[103] διπλάσον J

they yield 3500. Then finally, since we ought to circumvent one, we say: raise 2500; and 1000 are left out, which is indeed one thousand of one.

**Problem e**. The riddle of the ring. A trivialized variant, in which one has to find the finger in which someone among several people hold a ring; people must be arranged in a circle and reckoned starting from the one who holds the ring. The final explanation is interesting since it involves factoring out ("circumvent") the unit. Problems **c** and **e** are of the same kind. *Equation*. $10\{10[5(2x + 5)] + y\} = 1000x + 10y + 2500 = k$. *Algorithm*. $(k) \rightarrow k - 2500 \rightarrow myr(k - 2500) = x \,.\, dec(k - 2500) = y$.

## f

[= *Anonymus* P, no. 111–112 = Vindob. phil. gr. 225, f. 154v]

<Ἐ>ρώτησις

<Κ>αβαλλάριοι ἑκατὸν διερχόμενοι εὗρον μηλέαν, καὶ ὁ μὲν πρῶτος ἁπλώσας εἰς τὴν μηλέαν ἀνελάβετο μῆλον ἕν, ὁ δεύτερος δύο, ὁ τρίτος τρία, ὁ τέταρτος τέσσαρα, ὁ πέμπτος ε, ὁ ἕκτος ς, ὁ ζος ζ, ὁ ὄγδοος η, ὁ ἔννατος θ, ὁ δέκατος ι, καὶ καθεξῆς ἕως τῶν ἑκατόν, καὶ ἐπλήρωσαν ἅπαντα τὰ μῆλα. δέον γνῶναι πόσα μῆλα εἶχεν ἡ μηλέα.

μέθοδος. <Π>ολλαπλασίασον τὰ ἑκατὸν ἐφ' ἑαυτά, εἰπὼν ρ ρ· α· πρόσθες ρ· ὁμοῦ μύρια ἑκατόν· τὰ 𐅵 τούτων· ͵εν. καὶ εἶχε ἡ μηλέα μῆλα ͵εν. καὶ ἐπὶ τῶν ἄλλων ὁμοίως.

ἐὰν δὲ ὁ πρῶτος ἀφείλετο δύο, ὁ δεύτερος τέσσαρα, ὁ τρίτος ς, ὁ τέταρτος ὀκτώ, ὁ πέμπτος δέκα, ὁ ςος δώδεκα, καὶ ἐπληρώθησαν τὰ μῆλα μέχρι τῶν ρ καβαλλαρίων, πόσα ἂν ἐβάσταζεν ἡ μηλέα μῆλα;

ποιῶμεν οὕτως. πολλαπλασιάζομεν τὰ ρ ἐφ' ἑαυτά[104], λέγοντες ρ ρ· α· καὶ προστίθεμεν εἰς τὰ μύρια ρ· ὁμοῦ μύρια ἑκατόν, ἅτινα μοιρασίαν οὐ δέχονται, ἀλλ' ἀπεντεῦθεν λέγομεν ὅτι τόσα μῆλα.

Question.

One hundred passing-through riders found an apple orchard, and the first breaking into the apple orchard took up one apple, the second two, the third three, the fourth four, the fifth 5, the sixth 6, the 7th 7, the eighth 8, the ninth 9, the tenth 10, and in succession as far as one hundred, and they cleared all apples up. One must know how many apples had the apple orchard.

Procedure. Multiply one hundred by itself, saying 100 ‹by› 100: 10000; add 100; together one myriad one hundred; ½ of these; 5050. And the apple orchard had 5050 apples. And similarly for the others.

And if the first removed two, the second four, the third 6, the fourth eight, the fifth ten, the sixth twelve, and the apples were cleared up as far as the one hundred riders, how many apples would the apple orchard have hold?

Let's do as follows. We multiply 100 by themselves, saying 100 ‹by› 100: 10000; and we add 100 to the myriad; together one myriad one hundred, which indeed do not receive a partition, but we thereby say that the apples are such.

**Problem f**. Sum of an arithmetic progression. A copying mistake is corrected on the basis of the other two witnesses of the problem. Byzantine parallels. Five short arithmetical texts are ascribed to Kydones and to Argyros (ed. Acerbi, I problemi aritmetici); three of them expound procedures, with different degrees of generality, for the sum of an arithmetic progression; a fourth provides a proof of one such procedure. Cf. *Anonymus* P, no. 23, 37, 110–113; *Anonymus* 1436, no. 57–60 (for no. 110, see also, at f. 208v of the same manuscript as *Anonymus* P, the text edited in *HOO* IV xvi.16–xvii.5—a comparison of the two versions in Acerbi, I problemi aritmetici, Text 22); Vindob. phil. gr. 225, f. 154v (cf. *HOO* V cvii); and Moschopoulos' treatise on magic squares (ed. P. Tannery, Le traité de Manuel

[104] ἕν J

Moschopoulos sur les carrés magiques. Texte grec et traduction. *Annuaire de l'Association pour l'encouragement des études grecques en France* (1886) 88–118, repr. Id., Mémoires scientifiques IV. Toulouse – Paris 1920, 27–60: 34.24–36.9; Vat. gr. 1411, f. 118v, has a text identical to Tannery's; this manuscript is the earliest witness of the treatise; on Moschopoulos see Tannery, Manuel Moschopoulos; cf. also *PLP*, no. 19373). Recall that a magic square is the arrangement, on the $n^2$ cells of a "chessboard", of the first $n^2$ integers so that the sum of the numbers in any row, column and in the two main diagonals is the same. Such a sum is equal to the sum of the $n^2$ arranged integers, divided by the number of rows (or columns), namely, by *n*. *Algorithm*. $(n) \rightarrow nn \rightarrow nn + n \rightarrow (nn + n)/2$.

## APPENDIX

The list of resolutions of common fractions into unit fractions in Par. gr. 1670, ff. 44v–46v (P) is here edited and translated in tabular form. The list starts with fifths in the manuscript; the reason must be that the set of fractions with denominations from 2 to 4 would provide empty or trivial sets of resolutions. Recall that $\frac{2}{3}$ counts as a "unit fraction".

τὰ πέμπτα

ε$^{ον}$ τοῦ ἑνός, ε$^{ον}$· τῶν β, γ$^{ον}$ ιε$^{ον}$ ἢ δ$^{ον}$ ι$^{ον}$ κ$^{ον}$· τῶν γ, 𐅵 ι$^{ον}$ ἢ γ$^{ον}$ ε$^{ον}$ ιε$^{ον}$ ἢ δ$^{ον}$ ε$^{ον}$ ι$^{ον}$ κ$^{ον}$· τῶν δ, 𐅵 ε$^{ον}$ ι$^{ον}$ ἢ 𐅷 ι$^{ον}$ λ$^{ον}$· τῶν ε, α.

### Fifths

| numerator | 2 | 3 | 4 |
|---|---|---|---|
| resolutions | $\frac{1}{3}$ $\frac{1}{15}$ | $\frac{1}{2}$ $\frac{1}{10}$ | $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{10}$ |
| | $\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{20}$ | $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{15}$ | $\frac{2}{3}$ $\frac{1}{10}$ $\frac{1}{30}$ |
| | | $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{20}$ | |

τὰ ἕκτα

ς$^{ον}$ τοῦ ἑνός, ς$^{ον}$· τῶν β, γ$^{ον}$· τῶν γ, 𐅵 ἢ γ$^{ον}$ ς$^{ον}$· τῶν δ, 𐅷 ἢ 𐅵 ς$^{ον}$· τῶν ε, 𐅷 ς$^{ον}$ ἢ 𐅵 γ$^{ον}$· τῶν ς, α.

### Sixths

| numerator | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| resolutions | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{2}{3}$ $\frac{1}{6}$ |
| | | $\frac{1}{3}$ $\frac{1}{6}$ | $\frac{1}{2}$ $\frac{1}{6}$ | $\frac{1}{2}$ $\frac{1}{3}$ |

τὰ ἕβδομα

ζ$^{ον}$ τοῦ ἑνός, ζ$^{ον}$· τῶν β, δ$^{ον}$ κη$^{ον}$ ἢ ε$^{ον}$ ιδ$^{ον}$ ο$^{ον}$ ἢ ς$^{ον}$ ιδ$^{ον}$ κα$^{ον}$· τῶν γ, δ$^{ον}$ ζ$^{ον}$ κη$^{ον}$ ἢ γ$^{ον}$ ιδ$^{ον}$ μβ$^{ον}$ ἢ γ$^{ον}$ ιε$^{ον}$ λε$^{ον}$ ἢ ε$^{ον}$ ζ$^{ον}$ ιδ$^{ον}$ ο$^{ον}$ ἢ ς$^{ον}$ ζ$^{ον}$ ιδ$^{ον}$ κα$^{ον}$· τῶν δ, 𐅵 ιδ$^{ον}$· τῶν ε, 𐅵 ζ$^{ον}$ ιδ$^{ον}$ ἢ 𐅷 κα$^{ον}$· τῶν ς, 𐅵 γ$^{ον}$ μβ$^{ον}$ ἢ 𐅷 ζ$^{ον}$ κα$^{ον}$· τῶν ζ, μία.

## Sevenths

| numerator | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| resolutions | $\frac{1}{4}\ \frac{1}{28}$ | $\frac{1}{4}\ \frac{1}{7}\ \frac{1}{28}$ | $\frac{1}{2}\ \frac{1}{14}$ | $\frac{1}{2}\ \frac{1}{7}\ \frac{1}{14}$ | $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{42}$ |
| | $\frac{1}{5}\ \frac{1}{14}\ \frac{1}{70}$ | $\frac{1}{3}\ \frac{1}{14}\ \frac{1}{42}$ | | $\frac{2}{3}\ \frac{1}{21}$ | $\frac{2}{3}\ \frac{1}{7}\ \frac{1}{21}$ |
| | $\frac{1}{6}\ \frac{1}{14}\ \frac{1}{21}$ | $\frac{1}{3}\ \frac{1}{15}\ \frac{1}{35}$ | | | |
| | | $\frac{1}{5}\ \frac{1}{7}\ \frac{1}{14}\ \frac{1}{70}$ | | | |
| | | $\frac{1}{6}\ \frac{1}{7}\ \frac{1}{14}\ \frac{1}{21}$ | | | |

τὰ ὄγδοα

η$^{ον}$ τοῦ ἑνός, η$^{ον}$· τῶν β, δ$^{ον}$· τῶν γ, δ$^{ον}$ η$^{ον}$ ἢ γ$^{ον}$ κδ$^{ον}$· τῶν δ, 𐅵· τῶν ε, 𐅵 η$^{ον}$· τῶν ς, 𐅵 δ$^{ον}$ ἢ 𐅷 ις$^{ον}$· τῶν ἑπτά, 𐅵 δ$^{ον}$ η$^{ον}$ ἢ 𐅷 η$^{ον}$ ις$^{ον}$ ἢ 𐅵 γ$^{ον}$ κδ$^{ον}$ ἢ 𐅷 ς$^{ον}$ κδ$^{ον}$· τῶν ὀκτώ, μία.

## Eights

| numerator | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| resolutions | $\frac{1}{4}$ | $\frac{1}{4}\ \frac{1}{8}$ | $\frac{1}{2}$ | $\frac{1}{2}\ \frac{1}{8}$ | $\frac{1}{2}\ \frac{1}{4}$ | $\frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}$ |
| | | $\frac{1}{3}\ \frac{1}{24}$ | | | $\frac{2}{3}\ \frac{1}{16}$ | $\frac{2}{3}\ \frac{1}{8}\ \frac{1}{16}$ |
| | | | | | | $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{24}$ |
| | | | | | | $\frac{2}{3}\ \frac{1}{6}\ \frac{1}{24}$ |

τὰ ἔννατα

θ$^{ον}$ τοῦ ἑνός, θ$^{ον}$· τῶν β, ς$^{ον}$ ιη$^{ον}$ ἢ ε$^{ον}$ με$^{ον}$· τῶν γ, γ$^{ον}$· τῶν δ, γ$^{ον}$ θ$^{ον}$· τῶν ε, 𐅵 ιη$^{ον}$· τῶν ς, 𐅵 ς$^{ον}$ ἢ 𐅷· τῶν ζ, 𐅵 ς$^{ον}$ θ$^{ον}$ ἢ 𐅷 θ$^{ον}$· τῶν ὀκτώ, 𐅵 γ$^{ον}$ ιη$^{ον}$ ἢ 𐅷 ς$^{ον}$ ιη$^{ον}$· |[45r] τῶν ἐννέα, μία.

## Ninths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| resolutions | $\frac{1}{6}\ \frac{1}{18}$ | $\frac{1}{3}$ | $\frac{1}{3}\ \frac{1}{9}$ | $\frac{1}{2}\ \frac{1}{18}$ | $\frac{1}{2}\ \frac{1}{6}$ | $\frac{1}{2}\ \frac{1}{6}\ \frac{1}{9}$ | $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{18}$ |
| | $\frac{1}{5}\ \frac{1}{45}$ | | | | $\frac{2}{3}$ | $\frac{2}{3}\ \frac{1}{9}$ | $\frac{2}{3}\ \frac{1}{6}\ \frac{1}{18}$ |

τὰ δέκατα

ι$^{ον}$ τοῦ ἑνός, ι$^{ον}$· τῶν β, ε$^{ον}$· τῶν γ, ε$^{ον}$ ι$^{ον}$ ἢ δ$^{ον}$ κ$^{ον}$· τῶν δ, γ$^{ον}$ ιε$^{ον}$ ἢ δ$^{ον}$ ι$^{ον}$ κ$^{ον}$· τῶν ε, 𐅵· τῶν ς, 𐅵 ι$^{ον}$· τῶν ζ, 𐅵 ε$^{ον}$ ἢ 𐅷 λ$^{ον}$· τῶν η, 𐅵 ε$^{ον}$ ι$^{ον}$ ἢ 𐅷 ι$^{ον}$ λ$^{ον}$· τῶν θ, 𐅵 γ$^{ον}$ ιε$^{ον}$ ἢ 𐅵 δ$^{ον}$ ι$^{ον}$ κ$^{ον}$ ἢ 𐅷 ε$^{ον}$ λ$^{ον}$ ἢ 𐅷 ς$^{ον}$ ιε$^{ον}$· τῶν ι, α.

## Tenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| resolutions | $\frac{1}{5}$ | $\frac{1}{5}\ \frac{1}{10}$ | $\frac{1}{3}\ \frac{1}{15}$ | $\frac{1}{2}$ | $\frac{1}{2}\ \frac{1}{10}$ | $\frac{1}{2}\ \frac{1}{5}$ | $\frac{1}{2}\ \frac{1}{5}\ \frac{1}{10}$ | $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{15}$ |
| | | $\frac{1}{4}\ \frac{1}{20}$ | $\frac{1}{4}\ \frac{1}{10}\ \frac{1}{20}$ | | | $\frac{2}{3}\ \frac{1}{30}$ | $\frac{2}{3}\ \frac{1}{10}\ \frac{1}{30}$ | $\frac{1}{2}\ \frac{1}{4}\ \frac{1}{10}\ \frac{1}{20}$ |
| | | | | | | | | $\frac{2}{3}\ \frac{1}{5}\ \frac{1}{30}$ |
| | | | | | | | | $\frac{2}{3}\ \frac{1}{6}\ \frac{1}{15}$ |

τὰ ἑνδέκατα

ια$^{ον}$ τοῦ ἑνός, ια$^{ον}$· τῶν β, ς$^{ον}$ ξς$^{ον}$· τῶν γ, δ$^{ον}$ μδ$^{ον}$· τῶν δ, γ$^{ον}$ λγ$^{ον}$· τῶν ε, γ$^{ον}$ ια$^{ον}$ λγ$^{ον}$· τῶν ς, 𐅵 κβ$^{ον}$· τῶν ζ, 𐅵 ια$^{ον}$ κβ$^{ον}$· τῶν η, 𐅷 κβ$^{ον}$ ξς$^{ον}$· τῶν ἐννέα, 𐅷 ια$^{ον}$ κβ$^{ον}$ ξς$^{ον}$ ἢ 𐅵 δ$^{ον}$ κβ$^{ον}$ μδ$^{ον}$· τῶν ι, 𐅷 ς$^{ον}$ κβ$^{ον}$ λγ$^{ον}$ ἢ 𐅵 γ$^{ον}$ κβ$^{ον}$ λγ$^{ον}$· τῶν ια, α.

## Elevenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| resolutions | 1/6 1/66 | 1/4 1/44 | 1/3 1/33 | 1/3 1/11 1/33 | 1/2 1/22 | 1/2 1/11 1/22 | 2/3 1/22 1/66 | 2/3 1/11 1/22 1/66 | 2/3 1/6 1/22 1/33 |
| | | | | | | | | 1/2 1/4 1/22 1/44 | 1/2 1/3 1/22 1/33 |

τὰ δωδέκατα

ιβ$^{ον}$ τοῦ ἑνός, ιβ$^{ον}$· τῶν β, ς$^{ον}$· τῶν γ, δ$^{ον}$ ἢ ς$^{ον}$ ις$^{ον}$· τῶν δ, γ$^{ον}$ ἢ δ$^{ον}$ ις$^{ον}$· τῶν ε, γ$^{ον}$ ις$^{ον}$ ἢ δ$^{ον}$ ς$^{ον}$· τῶν ς, 𐅵 ἢ γ$^{ον}$ ς$^{ον}$· τῶν ζ, 𐅵 ις$^{ον}$ ἢ γ$^{ον}$ δ$^{ον}$· τῶν η, 𐅵 ς$^{ον}$ ἢ 𐅷· τῶν ἐννέα, 𐅵 δ$^{ον}$ ἢ 𐅷 ις$^{ον}$· τῶν ι, 𐅵 γ$^{ον}$ ἢ 𐅷 ς$^{ον}$· τῶν ια, 𐅵 γ$^{ον}$ ις$^{ον}$ ἢ 𐅷 δ$^{ον}$· τῶν δώδεκα, μία.

## Twelfths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| resolutions | 1/6 | 1/4 | 1/3 | 1/3 1/16 | 1/2 | 1/2 1/16 | 1/2 1/6 | 1/2 1/4 | 1/2 1/3 | 1/2 1/3 1/16 |
| | | 1/6 1/16 | 1/4 1/16 | 1/4 1/6 | 1/3 1/6 | 1/3 1/4 | 2/3 | 2/3 1/16 | 2/3 1/6 | 2/3 1/4 |

τὰ τρισκαιδέκατα

ιγ$^{ον}$ τοῦ ἑνός, ιγ$^{ον}$· τῶν β, ζ$^{ον}$ ϙα$^{ον}$· τῶν γ, ζ$^{ον}$ ιγ$^{ον}$ ϙα$^{ον}$ ἢ ς$^{ον}$ κς$^{ον}$ λθ$^{ον}$ ἢ ε$^{ον}$ λθ$^{ον}$ ρϙε$^{ον}$· τῶν δ, κς$^{ον}$ νβ$^{ον}$ ἢ ε$^{ον}$ ιγ$^{ον}$ λθ$^{ον}$ ρϙε$^{ον}$ ἢ ς$^{ον}$ ιγ$^{ον}$ κς$^{ον}$ λθ$^{ον}$· τῶν ε, γ$^{ον}$ κς$^{ον}$ οη$^{ον}$ ἢ δ$^{ον}$ ιγ$^{ον}$ κς$^{ον}$ νβ$^{ον}$· τῶν ς, γ$^{ον}$ ιγ$^{ον}$ κς$^{ον}$ οη$^{ον}$· τῶν ζ, 𐅵 κς$^{ον}$· τῶν ὀκτώ, 𐅵 ιγ$^{ον}$ κς$^{ον}$· τῶν θ, 𐅷 λθ$^{ον}$· τῶν ι, 𐅷 ιγ$^{ον}$ λθ$^{ον}$ ἢ 𐅵 δ$^{ον}$ νβ$^{ον}$· τῶν ια, 𐅵 γ$^{ον}$ οη$^{ον}$ ἢ 𐅷 ς$^{ον}$ οη$^{ον}$· τῶν ιβ, 𐅵 γ$^{ον}$ ιγ$^{ον}$ οη$^{ον}$ ἢ 𐅷 ς$^{ον}$ ιγ$^{ον}$ οη$^{ον}$ ἢ 𐅷 δ$^{ον}$ ρνς$^{ον}$· τῶν ιγ, μία.

## Thirteenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| resolutions | 1/7 1/91 | 1/7 1/13 1/91 | 1/26 1/52 | 1/3 1/26 1/78 | 1/3 1/13 1/26 1/78 | 1/2 1/26 |
| | | 1/6 1/26 1/39 | 1/5 1/13 1/39 1/195 | 1/4 1/13 1/26 1/52 | | |
| | | 1/5 1/39 1/195 | 1/6 1/13 1/26 1/39 | | | |

| numerator | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|
| resolutions | 1/2 1/13 1/26 | 2/3 1/39 | 2/3 1/13 1/39 | 1/2 1/3 1/78 | 1/2 1/3 1/13 1/78 |
| | | | 1/2 1/4 1/52 | 2/3 1/6 1/78 | 2/3 1/6 1/13 1/78 |
| | | | | | 2/3 1/4 1/156 |

τὰ τεσσαρεσκαιδέκατα

ιδ$^{\text{ον}}$ τοῦ ἑνός, ιδ$^{\text{ον}}$· τῶν β, ζ$^{\text{ον}}$· τῶν γ, ζ$^{\text{ον}}$ ιδ$^{\text{ον}}$· τῶν δ, δ$^{\text{ον}}$ κη$^{\text{ον}}$ ἢ ς$^{\text{ον}}$ ιδ$^{\text{ον}}$ κα$^{\text{ον}}$· τῶν ε, δ$^{\text{ον}}$ ιδ$^{\text{ον}}$ κη$^{\text{ον}}$ |[45v] ἢ γ$^{\text{ον}}$ μβ$^{\text{ον}}$ ἢ ς$^{\text{ον}}$ ζ$^{\text{ον}}$ κα$^{\text{ον}}$· τῶν ς, δ$^{\text{ον}}$ ζ$^{\text{ον}}$ κη$^{\text{ον}}$ ἢ γ$^{\text{ον}}$ ιδ$^{\text{ον}}$ μβ$^{\text{ον}}$ ἢ ς$^{\text{ον}}$ ζ$^{\text{ον}}$ ιδ$^{\text{ον}}$ κα$^{\text{ον}}$· τῶν ζ, 𐅵· τῶν η, 𐅵 ιδ$^{\text{ον}}$· τῶν θ, 𐅵 ζ$^{\text{ον}}$· τῶν ι, 𐅵 ζ$^{\text{ον}}$ ιδ$^{\text{ον}}$ ἢ ω κα$^{\text{ον}}$· τῶν ια, 𐅵 δ$^{\text{ον}}$ κη$^{\text{ον}}$ ἢ ω ιδ$^{\text{ον}}$ κα$^{\text{ον}}$· τῶν ιβ, 𐅵 γ$^{\text{ον}}$ μβ$^{\text{ον}}$ ἢ ω ζ$^{\text{ον}}$ κα$^{\text{ον}}$· τῶν ιγ, 𐅵 γ$^{\text{ον}}$ ιδ$^{\text{ον}}$ μβ$^{\text{ον}}$ ἢ ω ζ$^{\text{ον}}$ ιδ$^{\text{ον}}$ κα$^{\text{ον}}$ ἢ ω δ$^{\text{ον}}$ πδ$^{\text{ον}}$· τῶν ιδ, μία.

## Fourteenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| resolutions | 1/7 | 1/7 1/14 | 1/4 1/28 | 1/4 1/14 1/28 | 1/4 1/7 1/28 | 1/2 |
| | | | 1/6 1/14 1/21 | 1/3 1/42 | 1/3 1/14 1/42 | |
| | | | | 1/6 1/7 1/21 | 1/6 1/7 1/14 1/21 | |

| numerator | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|
| resolutions | 1/2 1/14 | 1/2 1/7 | 1/2 1/7 1/10 | 1/2 1/4 1/28 | 1/2 1/3 1/42 | 1/2 1/3 1/14 1/42 |
| | | | 2/3 1/21 | 2/3 1/14 1/21 | 2/3 1/7 1/21 | 2/3 1/7 1/14 1/21 |
| | | | | | | 2/3 1/4 1/84 |

τὰ πεντεκαιδέκατα

ιε$^{\text{ον}}$ τοῦ ἑνός, ιε$^{\text{ον}}$· τῶν β, ι$^{\text{ον}}$ λ$^{\text{ον}}$ ἢ η$^{\text{ον}}$ ρκ$^{\text{ον}}$ ἢ θ$^{\text{ον}}$ με$^{\text{ον}}$· τῶν γ, ε$^{\text{ον}}$ ἢ ι$^{\text{ον}}$ ιε$^{\text{ον}}$ λ$^{\text{ον}}$· τῶν δ, ε$^{\text{ον}}$ ιε$^{\text{ον}}$ ἢ δ$^{\text{ον}}$ ξ$^{\text{ον}}$· τῶν ε, γ$^{\text{ον}}$· τῶν ς, γ$^{\text{ον}}$ ιε$^{\text{ον}}$· τῶν ἑπτά, γ$^{\text{ον}}$ ι$^{\text{ον}}$ λ$^{\text{ον}}$ ἢ γ$^{\text{ον}}$ η$^{\text{ον}}$ ρκ$^{\text{ον}}$ ἢ γ$^{\text{ον}}$ θ$^{\text{ον}}$ με$^{\text{ον}}$· τῶν η, γ$^{\text{ον}}$ ε$^{\text{ον}}$ ἢ 𐅵 λ$^{\text{ον}}$· τῶν θ, 𐅵 ι$^{\text{ον}}$· τῶν ι, ω ἢ 𐅵 ι$^{\text{ον}}$ ιε$^{\text{ον}}$· τῶν ια, 𐅵 ε$^{\text{ον}}$ λ$^{\text{ον}}$ ἢ ω ιε$^{\text{ον}}$· τῶν ιβ, 𐅵 ε$^{\text{ον}}$ ι$^{\text{ον}}$ ἢ ω ι$^{\text{ον}}$ λ$^{\text{ον}}$· τῶν ιγ, 𐅵 ε$^{\text{ον}}$ ι$^{\text{ον}}$ ιε$^{\text{ον}}$ ἢ 𐅵 γ$^{\text{ον}}$ λ$^{\text{ον}}$ ἢ ω ε$^{\text{ον}}$· τῶν ιδ, 𐅵 γ$^{\text{ον}}$ ι$^{\text{ον}}$ ἢ ω ε$^{\text{ον}}$ ιε$^{\text{ον}}$· τῶν ιε, α.

## Fifteenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| resolutions | 1/10 1/30 | 1/5 | 1/5 1/15 | 1/3 | 1/3 1/15 | 1/3 1/10 1/30 | 1/3 1/5 | 1/2 1/10 |
| | 1/8 1/120 | 1/10 1/15 1/30 | 1/4 1/60 | | | 1/3 1/8 1/120 | 1/2 1/30 | |
| | 1/9 1/45 | | | | | 1/3 1/9 1/45 | | |

| numerator | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| resolutions | 2/3 | 1/2 1/5 1/30 | 1/2 1/5 1/10 | 1/2 1/5 1/10 1/15 | 1/2 1/3 1/10 |
| | 1/2 1/10 1/15 | 2/3 1/15 | 2/3 1/10 1/30 | 1/2 1/3 1/30 | 2/3 1/5 1/15 |
| | | | | 2/3 1/5 | |

τὰ ἑξκαιδέκατα

ις$^{\text{ον}}$ τοῦ ἑνός, ις$^{\text{ον}}$· τῶν β, η$^{\text{ον}}$· τῶν γ, η$^{\text{ον}}$ ις$^{\text{ον}}$· τῶν δ, δ$^{\text{ον}}$· τῶν ε, δ$^{\text{ον}}$ ις$^{\text{ον}}$· τῶν ς, δ$^{\text{ον}}$ η$^{\text{ον}}$· τῶν ζ, δ$^{\text{ον}}$ η$^{\text{ον}}$ ις$^{\text{ον}}$· τῶν η, 𐅵· τῶν θ, 𐅵 ις$^{\text{ον}}$· τῶν ι, 𐅵 η$^{\text{ον}}$· τῶν ια, 𐅵 η$^{\text{ον}}$ ις$^{\text{ον}}$· τῶν ιβ, 𐅵 δ$^{\text{ον}}$· τῶν ιγ, 𐅵 δ$^{\text{ον}}$ ις$^{\text{ον}}$· τῶν ιδ, 𐅵 δ$^{\text{ον}}$ η$^{\text{ον}}$· τῶν ιε, 𐅵 δ$^{\text{ον}}$ η$^{\text{ον}}$ ις$^{\text{ον}}$· τῶν ις, α.

## Sixteenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| resolutions | 1/8 | 1/8 1/16 | 1/4 | 1/4 1/16 | 1/4 1/8 | 1/4 1/8 1/16 | 1/2 | 1/2 1/16 | 1/2 1/8 |

| numerator | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|
| resolutions | 1/2 1/8 1/16 | 1/2 1/4 | 1/2 1/4 1/16 | 1/2 1/4 1/8 | 1/2 1/4 1/8 1/16 |

τὰ ἑπτακαιδέκατα

ιζ$^{ον}$ τοῦ ἑνός, ιζ$^{ον}$· τῶν β, θ$^{ον}$ ρνγ$^{ον}$· τῶν γ, θ$^{ον}$ ιζ$^{ον}$ ρνγ$^{ον}$ ἢ ϛ$^{ον}$ ρβ$^{ον}$· τῶν δ, ϛ$^{ον}$ ιζ$^{ον}$ ρβ$^{ον}$ ἢ ε$^{ον}$ λδ$^{ον}$ ρο$^{ον}$· τῶν ε, δ$^{ον}$ λδ$^{ον}$ ξη$^{ον}$· τῶν ϛ, γ$^{ον}$ να$^{ον}$· τῶν ζ, γ$^{ον}$ ιζ$^{ον}$ να$^{ον}$· τῶν η, γ$^{ον}$ θ$^{ον}$ να$^{ον}$ ρνγ$^{ον}$· τῶν ἐννέα, 𐅵 λδ$^{ον}$· τῶν ι, 𐅵 ιζ$^{ον}$ λδ$^{ον}$· τῶν ια, 𐅵 θ$^{ον}$ λδ$^{ον}$ ρνγ$^{ον}$· τῶν ιβ, 𐅷 λδ$^{ον}$ ρβ$^{ον}$· τῶν ιγ, 𐅷 ιζ$^{ον}$ λδ$^{ον}$ ρβ$^{ον}$ ἢ 𐅷 ις$^{ον}$ ξη$^{ον}$ ἢ 𐅵 δ$^{ον}$ ξη$^{ον}$· τῶν ιδ, 𐅷 ιβ$^{ον}$[105] ιζ$^{ον}$ ξη$^{ον}$ ἢ 𐅵 δ$^{ον}$ ιζ$^{ον}$ ξη$^{ον}$· τῶν ιε, |[46r] 𐅷 ϛ$^{ον}$ λδ$^{ον}$ να$^{ον}$ ἢ 𐅵 γ$^{ον}$ λδ$^{ον}$ να$^{ον}$· τῶν ις, 𐅷 ϛ$^{ον}$ ιζ$^{ον}$ λδ$^{ον}$ να$^{ον}$ ἢ 𐅷 δ$^{ον}$ ξη$^{ον}$ ρβ$^{ον}$ ἢ 𐅵 γ$^{ον}$ ιζ$^{ον}$ λδ$^{ον}$ να$^{ον}$· τῶν δεκαεπτά, μία.

## Seventeenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| resolutions | 1/9 1/153 | 1/9 1/17 1/153 | 1/6 1/17 1/102 | 1/4 1/34 1/68 | 1/3 1/51 | 1/3 1/17 1/51 | 1/3 1/9 1/51 1/153 | 1/2 1/34 | 1/2 1/17 1/34 |
| | | 1/6 1/102 | 1/5 1/34 1/170 | | | | | | |

| numerator | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|
| resolutions | 1/2 1/9 1/34 1/153 | 2/3 1/34 1/102 | 2/3 1/17 1/34 1/102 | 2/3 1/12 1/17 1/68 | 2/3 1/6 1/34 1/51 | 2/3 1/6 1/17 1/34 1/51 |
| | | | 2/3 1/16 1/68 | 1/2 1/4 1/17 1/68 | 1/2 1/3 1/34 1/51 | 2/3 1/4 1/68 1/102 |
| | | | 1/2 1/4 1/68 | | | 1/2 1/3 1/17 1/34 1/51 |

τὰ ὀκτωκαιδέκατα

ιη$^{ον}$ τοῦ ἑνός, ιη$^{ον}$· τῶν β, θ$^{ον}$· τῶν γ, ϛ$^{ον}$· τῶν δ, ϛ$^{ον}$ ιη$^{ον}$· τῶν ε, ϛ$^{ον}$ θ$^{ον}$ ἢ δ$^{ον}$ λϛ$^{ον}$· τῶν ϛ, γ$^{ον}$· τῶν ζ, γ$^{ον}$ ιη$^{ον}$· τῶν η, γ$^{ον}$ θ$^{ον}$· τῶν θ, 𐅵· τῶν ι, 𐅵 ιη$^{ον}$· τῶν ια, 𐅵 θ$^{ον}$· τῶν ιβ, 𐅷· τῶν ιγ, 𐅷 ιη$^{ον}$· τῶν ιδ, 𐅷 θ$^{ον}$· τῶν ιε, 𐅷 ϛ$^{ον}$ ἢ 𐅵 γ$^{ον}$· τῶν ις, 𐅷 ϛ$^{ον}$ ιη$^{ον}$ ἢ 𐅵 γ$^{ον}$ ιη$^{ον}$· τῶν ιζ, 𐅷 ϛ$^{ον}$ θ$^{ον}$ ἢ 𐅵 γ$^{ον}$ θ$^{ον}$· τῶν δεκαοκτώ, μία.

## Eighteenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| resolutions | 1/9 | 1/6 | 1/6 1/18 | 1/6 1/9 | 1/3 | 1/3 1/18 | 1/3 1/9 | 1/2 | 1/2 1/18 | 1/2 1/9 |
| | | | | 1/4 1/36 | | | | | | |

| numerator | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|
| resolutions | 2/3 | 2/3 1/18 | 2/3 1/9 | 2/3 1/6 | 2/3 1/6 1/18 | 2/3 1/6 1/9 |
| | | | | 1/2 1/3 | 1/2 1/3 1/18 | 1/2 1/3 1/9 |

τὰ ἐννεακαιδέκατα

ιθ$^{ον}$ τοῦ ἑνός, ιθ$^{ον}$· τῶν β, ι$^{ον}$ ρϙ$^{ον}$· τῶν γ, ι$^{ον}$ ιθ$^{ον}$ ρϙ$^{ον}$ ἢ θ$^{ον}$ λη$^{ον}$ νζ$^{ον}$ τμβ$^{ον}$ ἢ η$^{ον}$ λη$^{ον}$ ρνβ$^{ον}$ ἢ ζ$^{ον}$ οϛ$^{ον}$ φλβ$^{ον}$· τῶν δ, ε$^{ον}$ ϙε$^{ον}$· τῶν ε, δ$^{ον}$ οϛ$^{ον}$· τῶν ϛ, δ$^{ον}$ ιθ$^{ον}$ οϛ$^{ον}$· τῶν ζ, δ$^{ον}$ ι$^{ον}$ οϛ$^{ον}$ ρϙ$^{ον}$ ἢ δ$^{ον}$ θ$^{ον}$ σκη$^{ον}$ τμβ$^{ον}$ ἢ γ$^{ον}$ λη$^{ον}$ ριδ$^{ον}$· τῶν η, δ$^{ον}$ ϛ$^{ον}$ σκη$^{ον}$ ἢ δ$^{ον}$ ι$^{ον}$ ιθ$^{ον}$ οϛ$^{ον}$ ρϙ$^{ον}$ ἢ γ$^{ον}$ ιθ$^{ον}$ λη$^{ον}$ ριδ$^{ον}$· τῶν θ, δ$^{ον}$ ε$^{ον}$ οϛ$^{ον}$ ϙε$^{ον}$ ἢ γ$^{ον}$ θ$^{ον}$ λη$^{ον}$ τμβ$^{ον}$ ἢ γ$^{ον}$ η$^{ον}$ οϛ$^{ον}$ υνϛ$^{ον}$· τῶν ι, 𐅵 λη$^{ον}$· τῶν ια, 𐅵 ιθ$^{ον}$ λη$^{ον}$· τῶν ιβ, 𐅵 η$^{ον}$ ρνβ$^{ον}$ ἢ 𐅵 ι$^{ον}$ λη$^{ον}$ ρϙ$^{ον}$· τῶν ιγ, 𐅷 νζ$^{ον}$ ἢ 𐅵 ϛ$^{ον}$ νζ$^{ον}$· τῶν ιδ, 𐅷 ιθ$^{ον}$ νζ$^{ον}$ ἢ 𐅵 ϛ$^{ον}$ ιθ$^{ον}$ νζ$^{ον}$ ἢ 𐅵 ε$^{ον}$ λη$^{ον}$ ϙε$^{ον}$· τῶν ιε, 𐅷 ι$^{ον}$ νζ$^{ον}$ ρϙ$^{ον}$ ἢ 𐅷 θ$^{ον}$ ριδ$^{ον}$ τμβ$^{ον}$ ἢ 𐅵 δ$^{ον}$ λη$^{ον}$ οϛ$^{ον}$· τῶν ις, 𐅷 ϛ$^{ον}$ ριδ$^{ον}$ ἢ 𐅵 γ$^{ον}$ ριδ$^{ον}$· τῶν ιζ, 𐅷 ϛ$^{ον}$ ιθ$^{ον}$ ριδ$^{ον}$ ἢ 𐅵 γ$^{ον}$ ιθ$^{ον}$ ριδ$^{ον}$ ἢ 𐅷 ε$^{ον}$ νζ$^{ον}$ ϙε$^{ον}$· τῶν ιη, 𐅷 δ$^{ον}$ νζ$^{ον}$ οϛ$^{ον}$ ἢ 𐅵 γ$^{ον}$ ιβ$^{ον}$[106] νζ$^{ον}$ οϛ$^{ον}$· τῶν δεκαεννέα, μία.

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[105] ιϛ P
[106] ιϛ P

## Nineteenths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| resolutions | 1/10 1/190 | 1/10 1/19 1/190 | 1/5 1/95 | 1/4 1/76 | 1/4 1/19 1/76 | 1/4 1/10 1/76 1/190 | 1/4 1/6 1/228 | 1/4 1/5 1/76 1/95 |
| | | 1/9 1/38 1/57 1/342 | | | | 1/4 1/9 1/228 1/342 | 1/4 1/10 1/19 1/76 1/190 | 1/3 1/9 1/38 1/342 |
| | | 1/8 1/38 1/152 | | | | 1/3 1/38 1/114 | 1/3 1/19 1/38 1/114 | 1/3 1/8 1/76 1/456 |
| | | 1/7 1/76 1/532 | | | | | | |

| numerator | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|
| resolutions | 1/2 1/38 | 1/2 1/19 1/38 | 1/2 1/8 1/152 | 2/3 1/57 | 2/3 1/19 1/57 | 2/3 1/10 1/57 1/190 |
| | | | 1/2 1/10 1/38 1/190 | 1/2 1/6 1/57 | 1/2 1/6 1/19 1/57 | 2/3 1/9 1/114 1/342 |
| | | | | | 1/2 1/5 1/38 1/95 | 1/2 1/4 1/38 1/76 |

| numerator | 16 | 17 | 18 |
|---|---|---|---|
| resolutions | 2/3 1/6 1/114 | 2/3 1/6 1/19 1/114 | 2/3 1/4 1/57 1/76 |
| | 1/2 1/3 1/114 | 1/2 1/3 1/19 1/114 | 1/2 1/3 1/12 1/57 1/76 |
| | | 2/3 1/5 1/57 1/95 | |

τὰ εἰκοστά

κ$^{ον}$ τοῦ ἑνός, κ$^{ον}$· τῶν β, ι$^{ον}$· τῶν γ, ι$^{ον}$ κ$^{ον}$· τῶν δ, ε$^{ον}$· |[46v] τῶν ε, δ$^{ον}$· τῶν ς, ε$^{ον}$ ι$^{ον}$ ἢ δ$^{ον}$ κ$^{ον}$· τῶν ζ, δ$^{ον}$ ι$^{ον}$· τῶν ὀκτώ, γ$^{ον}$ ιε$^{ον}$· τῶν ἐννέα, γ$^{ον}$ ιε$^{ον}$ κ$^{ον}$ ἢ δ$^{ον}$ ε$^{ον}$· τῶν ι, 𐅵· τῶν ια, 𐅵 κ$^{ον}$· τῶν ιβ, 𐅵 ι$^{ον}$· τῶν ιγ, 𐅵 ι$^{ον}$ κ$^{ον}$ ἢ γ$^{ον}$ δ$^{ον}$ ιε$^{ον}$· τῶν ιδ, 𐅵 ε$^{ον}$ ἢ 𐅷 λ$^{ον}$· τῶν ιε, 𐅵 δ$^{ον}$ ἢ 𐅵 ε$^{ον}$ κ$^{ον}$ ἢ 𐅷 ις$^{ον}$· τῶν ις, 𐅵 ε$^{ον}$ ι$^{ον}$ ἢ 𐅷 ι$^{ον}$ λ$^{ον}$· τῶν ιζ, 𐅵 δ$^{ον}$ ι$^{ον}$ ἢ 𐅷 ς$^{ον}$ ξ$^{ον}$ ἢ 𐅷 ι$^{ον}$ ις$^{ον}$· τῶν ιη, 𐅵 γ$^{ον}$ ιε$^{ον}$ ἢ 𐅵 δ$^{ον}$ ι$^{ον}$ κ$^{ον}$ ἢ 𐅷 ε$^{ον}$ λ$^{ον}$ ἢ 𐅷 ς$^{ον}$ ιε$^{ν}$· τῶν ιθ, 𐅵 δ$^{ον}$ ε$^{ον}$ ἢ 𐅵 γ$^{ον}$ ιε$^{ον}$ κ$^{ον}$ ἢ 𐅷 ε$^{ον}$ ις$^{ον}$ ἢ 𐅷 ς$^{ον}$ ιε$^{ον}$ κ$^{ον}$ ἢ 𐅷 ς$^{ον}$ ι$^{ον}$ ξ$^{ον}$· τῶν εἴκοσι, μία.

## Twentieths

| numerator | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| resolutions | 1/10 | 1/10 1/20 | 1/5 | 1/4 | 1/5 1/10 | 1/4 1/10 | 1/3 1/5 | 1/3 1/15 1/20 | 1/2 | 1/2 1/20 |
| | | | | | 1/4 1/20 | | | 1/4 1/5 | | |

| numerator | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|
| resolutions | 1/2 1/10 | 1/2 1/10 1/20 | 1/2 1/5 | 1/2 1/4 | 1/2 1/5 1/10 | 1/2 1/4 1/10 | 1/2 1/3 1/15 | 1/2 1/4 1/5 |
| | | 1/3 1/4 1/15 | 2/3 1/30 | 1/2 1/5 1/20 | 2/3 1/10 1/30 | 2/3 1/6 1/60 | 1/2 1/4 1/10 1/20 | 1/2 1/3 1/15 1/20 |
| | | | | 2/3 1/16 | | 2/3 1/10 1/16 | 2/3 1/5 1/30 | 2/3 1/5 1/16 |
| | | | | | | | 2/3 1/6 1/15 | 2/3 1/6 1/15 1/20 |
| | | | | | | | | 2/3 1/6 1/10 1/60 |

The method expounded in Par. gr. 1670 to resolve common fractions into unit fractions is as follows; I take the resolution of 5/7 on f. 40v as an example:

ζ$^{ον}$ τῶν πέντε, 𐅵 ζ$^{ον}$ ιδ$^{ον}$. ἡ μέθοδος. ἀνάλυσον τὰς πέντε μονάδας εἰς ἡμίσεια· γίνονται ἡμίσεια δέκα, ἀφ’ ὧν ἐπίδος τοῖς ἑπτὰ ἀνὰ ἥμισυ, ἤτοι ἡμίσεια ἑπτά· λοιπὰ ἡμίσεια τρία, ἤτοι μονὰς μία ἥμισυ. ἀνάλυσον οὖν τὴν μονάδα εἰς ζζ$^{α}$, καὶ ἐπίδος τοῖς ἑπτὰ ἀνὰ ζ$^{ον}$· τὸ δὲ ἥμισυ πολυπλασίασον ἐπὶ τὰ ἑπτὰ οὕτως. β ζ· ιδ, ὧν τὸ ἥμισυ γίνεται ιδιδ$^{α}$ ἑπτά, καὶ ἐπίδος τοῖς ἑπτὰ ἀνὰ ιδ$^{ον}$. γίνεται οὖν ὁ μερισμὸς τῶν πέντε εἰς ἑπτὰ ∠ ζ$^{ον}$ ιδ$^{ον}$. καὶ εἰπὲ οὕτως. ἑπτάκις τὸ ἥμισυ ἑπτὰ ἡμίσεια, ἤτοι μονάδες γ ∠· ἑπτάκις τὸ ζ$^{ον}$ ἑπτὰ ζζ$^{α}$, ἤτοι μονὰς μία· καὶ ἑπτάκις τὸ ιδ$^{ον}$ ἑπτὰ ιδιδ$^{α}$, ἤτοι ἥμισυ τῆς μονάδος.

1/7 of five, 1/2 1/7 1/14. Procedure. Resolve the five units into halves; they yield ten halves, from which give a half to each seven, that is, seven halves; three halves as remainders, that is, one unit and a half. Then resolve the unit into sevenths, and give a 1/7 to each seven; and multiply the half by seven as follows. 2 ‹by› 7: 14, a half of which yields seven sevenths, and give a 1/14 to each seven. Then the division of five into seven yields 1/2 1/7 1/14. And say as follows. Seven times a half seven halves, that is, 3 1/2 units; seven times 1/7 seven sevenths, that is, one unit; and seven times 1/14 seven fourteenths, that is, a half of a unit.

A procedure like this seems to presuppose the result, but this is not the case, for what is required is to write a common fraction as a sum of unit fractions. Let us consider the greatest unit fraction in any resolution. Now, neglecting for simplicity 2/3, by definition such a fraction cannot be greater than 1/2, and stricter upper bounds can easily be set in specific cases. On the other hand, it is easy to see that the denomination of the greatest unit fraction in any "reasonable" resolution cannot be equal to or greater than the denomination of the common fraction to be resolved. For instance, a "reasonable" resolution of 3/7 cannot have 1/7 or 1/8 as its greatest unitary fraction. Thus, the denomination of the greatest unit fraction in any resolution of a common fraction with denomination 7 can only be 2, 3, 4, 5, or 6. We may now apply uniformly the algorithm of our text, which can be described in modern fashion as follows.

The numerator of the common fraction to be resolved is rescaled into an equivalent fraction whose denomination is one of the possible values. To be consistent with our example, select 2 as such a denomination and write 5 → 10/2. Write this fraction as sum of two fractions, the first of which has a numerator that is a multiple of the denomination of the common fraction at issue, here 7: 5 → 10/2 → 7/2 + 3/2. The second fraction is either an improper fraction, or a common fraction, or 2/3, or a unit fraction. If the second or the third case apply, resolve into unit fractions (use 2/3 = 1/2 + 1/6)—this is always possible since the second fraction necessarily has a denomination less than the one of the fraction to be resolved, and since the resolutions are computed serially and by increasing denominations. If the first case applies, write the improper fraction as integral part + fractional part: 5 → 10/2 → 7/2 + 3/2 → 7/2 + 1 + 1/2. Treat 1 as the fraction 1/1 and, if the case applies, resolve the said fractional part into unit fractions: 5 → 10/2 → 7/2 + 3/2 → 7/2 + 1/1 + 1/2. Write the result—which contains only unit fractions with the sole exception of the first fraction set out in the second step of the algorithm—by factoring out the denomination of the fraction to be resolved, possibly after rescaling the fractions involved by the same denomination: 5 → 10/2 → 7/2 + 3/2 → 7/2 + 1/1 + 1/2 → 7(1/2) + 7(1/7) + 7(1/14). If all fractions involved in the last step are unit fractions, their sum is the required resolution and the algorithm ends: 5/7 = 1/2 + 1/7 + 1/14. If they are not—and this can only happen if the first fraction in the second step of the algorithm yields, after factoring out the denomination of the fraction to be resolved, a common fraction—resolve the said common fraction into unit fractions.

This procedure is used in such a way as to yield resolutions that keep the number of unit fractions to a reasonable minimum. For instance, the table for the "Sevenths" above shows that further resolutions of 5/7 could be obtained by wildly combining those of 2/7 and those of 3/7, but this move is never put into effect. Note, however, that three of the five resolutions of 3/7 are simply obtained by adding the unit fraction 1/7 to the three resolutions of 2/7.

## LIST OF THE MANUSCRIPTS MENTIONED IN THE ARTICLE AND THEIR *DIKTYON* NUMBERS.

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- Pal. gr. 367 (*Diktyon* 66099)
- Ross. 986 (*Diktyon* 66453)
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- suppl. gr. 387 (*Diktyon* 53135)
- suppl. gr. 652 (*Diktyon* 53387)
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- suppl. gr. 920 (*Diktyon* 53604)

Uppsala, Universitets Bibliotek
- gr. 8 (*Diktyon* 64421)

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